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Effect of Returns to Scale in Different DEA Models on Evaluating Efficiency by Considering Uncertainty in Data: Application from Hospitals

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Abstract

One of the most essential methods for assessing the efficiency of Decision-Making Units (DMUs) is Data Envelopment Analysis (DEA). This method is nonparametric, and one of the most critical issues is considering uncertain data in evaluating and ranking DMUs. Robust Data Envelopment Analysis (RDEA) is the approach for measuring the relative efficiency of DMUs by considering uncertain data. In this paper, we developed a RDEA on the Variable Returns to Scale (VRS) approaches and compared the results of RDEA based on the BCC model with RDEA based on the CCR model of DEA. By using Robust optimization, we wrote the RDEA. For the Robust optimization, two approaches are introduced: One is the Ben-Tal and Nemirovski approach [1], [2], and the other is the Bertsimas et al. approach. In this paper, we used the Bertsimas et al. approach because this approach, unlike the Ben-Tal and Nemirovski approach [1], [2], is a linear programming problem and is not hard to solve.

Keywords: Data envelopment analysis, Robust optimization, Robust data envelopment analysis, Constant returns to scale, Variable returns to scale.

1 | Introduction

During the past few years, there have been various methods and techniques developed to estimate the efficiency scores of different Decision Making Units (DMUs) such as electricity distribution units, hospitals, universities, telecommunication companies, etc. These methods are generally classified as deterministic and stochastic methods. In the case of the deterministic, no errors in the data are assumed as statistical noise, but

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an error term is considered as the statistical noise for the stochastic methods. Also, one can classify the methods as parametric or non-parametric. In the parametric methods, a cost or production function is estimated, whereas in the non-parametric methods, it is not necessary to estimate the cost or production function. Corrected Ordinary Least Squares (COLS) and Stochastic Frontier Analysis (SFA) are parametric models, and Data Envelopment Analysis (DEA) and Principal Component Analysis (PCA) are considered to be non-parametric models. In addition, COLS, DEA, and PCA are typically considered to be deterministic, and SFA is considered to be stochastic [3].

DEA, among many researchers, has been widely used, and there are two general DEA methods: The Constant Returns to Scale (CRS) and the Variable Returns To Scale (VRS) approaches. The CRS hypothesis suggests that companies are flexible and adjust their size to the optimal firm size. In contrast, the VRS approach is less restrictive since it compares the efficiency of companies only within similar sample sizes. This approach is adopted if the companies are not free to choose or adapt their size. The comparison between the two approaches also provides some information about the underlying technology; if the results of the CRS and the VRS approaches are similar, then returns to scale do not play an essential role in the process [3]. There are also many real-world applications of the DEA method in different industries. For instance, Sadjadi and Omrani [4] applied Robust Data Envelopment Analysis (RDEA) for checking uncertainty in the data. They examined both Robust methods based on Ben-Tal and Nemirovski [1] and Bertsimas and Sim [5] to check uncertainty for two applications from the energy and telecommunication industries and analyzed their results. Roghanian and Foroughi [6] Applied RDEA for the Airport industry in Iran, and by using different input/output, they have examined DEA for major Iranian airports to measure the relative efficiencies of various airports. Khaki et al. [7] proposed a Robust methodology for DEA to measure the efficiency of health care systems, considering uncertainty on output parameters. Gharakhani et al. [8] developed a RDEA to measure the efficiency of high schools considering uncertainty on output parameters. They present an empirical study on a set of high schools located in Tehran, which is the capital city of Iran.

2 | Problem Statement

DEA is a nonparametric linear programming technique where a set of units is evaluated according to their input consumption and output production [9] since the seminal paper by Charnes et al. [9], a variety of DEA models have appeared in the literature. Two of the DEA models that are most often associated with the DEA methodology are the CCR and BCC models.

Let x_{ij} be the inputs for a decision unit with i=1,…,m and y_{rj} be the outputs with r=1,…,s and j=1,…,n. Let u_i and v_j be the dual variables associated with x_i and y_j, respectively. The linear programming statement for the (Input-oriented) CCR model is:

$$\begin{aligned} &\max z = \sum_{r=1}^{s} u_r\, y_{ro}\,, \\ &\text{s.t.} \\ &\sum_{i=1}^{m} v_i\, x_{io} = 1, \\ &\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0, \quad j = 1, \dots, n, \\ &u_r, v_i \ge 0. \end{aligned} \tag{1}$$

The dual (Multiplier) form of the BCC model is formulated as follows [10]:

$$\max z = \sum_{r=1}^{s} u_r y_{ro} + d_0,$$

$$\text{s.t.}$$

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_0 \le 0, \tag{2}$$

$u_r, v_i \ge \varepsilon$, d_0 is free.

When there is uncertainty with the inputs and the outputs, we must use different techniques to make sure that a small change in input/output data does not change the output rankings. Thus, we use the RDEA to handle the uncertainty of data.

2.1 | Robust Optimization

In classical optimization modeling, input parameters are considered as certain values. However, in real cases, we are not certain about all parameter values. Robust optimization is a new approach to incorporate uncertainty within mathematical models. The approach based on Robust optimization is the most preferred method among practitioners due to its applicability. Recently, Robust optimization is very popular among practitioners and is applied in different contexts [7].

In classical modeling, a full probabilistic characterization is assumed under uncertainty. However, a representative nominal value is used instead of uncertainty, which is ignored in many models. Stochastic Programming (SP) is the classical approach to handling uncertainty. Recently, Robust optimization has been developed, which is considered an alternative to sensitivity analysis and SP [8].

Ben-Tal and Nemirovski [2] and Bertsimas and Sim [11] to handle uncertainty in data created new Robust optimization approaches. To present the Robust structure proposed by Ben-Tal and Nemirovski [2] and Bertsimas and Sim [11], assume the following LP problem:

$$\min c'x,$$

$$\text{s.t.}$$

$$Ax \ge b,$$

$$x \epsilon X. \tag{3}$$

By considering that the uncertainty influences the constraints A, in a Robust optimization approach, for expressing uncertainty in data, we consider a particular row i of the matrix A and assume the J_i is the set of coefficients in row i that lead to uncertainty in the data. Each input data by an uncertainty value, ($\tilde{a}_{ij}$) is expressed as follows:

($\tilde{a}_{ij}$) take values in [a_{ij} -$\hat{a}_{ij}$, a_{ij} +$\hat{a}_{ij}$], a_{ij} is the nominal value and $\hat{a}_{ij}$ is called the precision of the estimate. In the next section, express the RDEA based on Bertsimas and Sim's [5, 11, 12] and Bertsimas et al. approach.

2.2 | Robust Data Envelopment Analysis Based on Bertsimas and Sim [5, 11, 12] and Bertsimas et al. approach

By considering J_i: Set of coefficients of uncertain data, ith row constraint, *Model (2)* reformulated as follows:

$$\min c'x,$$

$$s.t.$$

$$\hat{a_{ij}}x \geq b, \quad (4)$$

$$x \in X.$$

Bertsimas and Thiele [13], [14] measures the deviation of parameter $\tilde{a_{ij}}$ from a_{ij} as follows:

$$\eta_{ij} = \frac{\tilde{a_{ij}} - a_{ij}}{\hat{a_{ij}}}, \quad (5)$$

$\tilde{a_{ij}}$ and a_{ij} are uncertain data and a nominal value, and $\hat{a_{ij}}$ measures the precision of the estimate. η_{ij} has an unknown but symmetric distribution that takes values in [-1,1]. Thus,

$\sum_{j=1}^{n} \eta_{ij}$ is in $[-n, n]$, $\sum_{j=1}^{n} \eta_{ij} \leq \Gamma_i$,

Γ_i is called the budget of uncertainty of constraint i and takes a value in $[0, n]$.

Γ_i adjust the Robustness of the proposed method against the level of conservatism of the solution.

There are three cases for Γ_i as follows:

I. If Γ_i=0, there is no protection against uncertainty.

II. If Γ_i=0, the ith constraint of the problem is completely protected against uncertainty.

III. If $\Gamma_i \epsilon$ (0,n), the decision-maker makes a trade-off between the protection level of the constraint and the degree of conservatism of the solution [4].

It is sufficient to choose Γ_i at least equal to $\Gamma_i = 1 + \Phi^{-1}(1 - e_i)\sqrt{n}$.

n is the number of uncertain parameters, and Φ is the CDF of a Gaussian distribution. However, for the case of our proposed method, since there are only four uncertain parameters, we choose Γ = 4 as recommended by Sadjadi and Omrani [4].

The Robust optimization based on Bertsimas and Sim [5], [11], [12], and Bertsimas and Sim [11] is as follows:

$$\min c'x,$$

$$s.t.$$

$$a_i'x - \Gamma_i p_i - \sum_{j \epsilon J_i} q_{ij} \geq 0,$$

$$p_i + q_{ij} \geq e a_{ij} y_j,$$

$$-y_j \leq x_j \leq y_j, \quad (6)$$

$$p_i, q_{ij} \geq 0,$$

$$x \epsilon X.$$

By considering *Model (1)* as linear programming and for uncertainty in outputs, use *Model (6)* and write the RDEA model based on Bertsimas et al.'s approach as follows [4]:

$$\max z,$$

$$s.t.$$

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{ro} - z_0 - \Gamma_0 p_0 - \sum_{j \epsilon J_i} q_{ro} \geq 0,$$

$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - \Gamma_j p_j - \sum_{j \epsilon J_i} q_{rj} \geq 0, \qquad j = 1, \ldots\ldots, n, \tag{7}$$

$$p_j + q_{rj} \geq e y_{rj} z_r, \qquad \text{for all } r, j,$$

$$-z_r \leq u_r \leq z_r, \qquad \text{for all } r,$$

$$p_j, q_{rj} \geq 0,$$

$$v_i, u_r \geq 0.$$

By considering *Model (2)* as linear programming and for uncertainty in outputs, we used *Model (6)* to write the RDEA model based on Bertsimas et al.'s approach as follows:

$$\max z,$$

$$\text{S. t.}$$

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{ro} - z_0 - \Gamma_0 p_0 - \sum_{j \epsilon J_i} q_{ro} + d_0 \geq 0,$$

$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - \Gamma_j p_j - \sum_{j \epsilon J_i} q_{rj} - d_0 \geq 0, \qquad j = 1, \ldots\ldots, n, \tag{8}$$

$$p_j + q_{rj} \geq e y_{rj} z_r, \qquad \text{for all } r, j,$$

$$-z_r \leq u_r \leq z_r, \qquad \text{for all } r,$$

$$p_j, q_{rj} \geq,$$

$$v_i, u_r \geq 0.$$

In *Models (7)* and *(8)*, x_{io} and y_{ro} are the ith input and rth output for the DMU under consideration. x_{ij} and y_{rj} are the i^{th} input and r^{th} output for the company j, also z_0 is the efficiency for the DMU under consideration. In the RDEA, e_i is the most probable for violating the constraint i that takes value in [0.001,0.1]. p_j, q_{rj} are the dual variables. We used the Software GAMS for solving these models.

3 | Case Study

In order to present a more detailed explanation of the above models, we used the actual data and compared the obtained results. In this paper, we considered 39 health centers and hospitals for evaluating and ranking them based on the CCR, BCC, and RDEA models. In *Table 1*, we show the inputs and outputs of 39 health centers and hospitals that are certain, and by considering uncertainty in data, we evaluate and rank them. Our aim in this paper is to check the effect of return to scale on evaluating efficiency and ranking them on the application of CCR and BCC models, and RDEA based on the CCR and BCC models of DEA. So in *Table 3,* we show the obtained results of evaluating efficiency and ranking the health centers and hospitals by considering certain and uncertain data based on the CCR model, and in *Table 4*, we show the obtained results of evaluating efficiency and ranking the health centers and hospitals by considering certain and uncertain data based on the BCC model.

Table 1. Input and output of the health centers.

	Health Centers	In1	In2	In3	In4	In5	Out1	Out2	Out3	Out4
1	Ghaem research and treatment center	34	36.3	36	44	35	6.25	11.3	5	20
2	Hashemi Nejad Hospital	24	23.8	37	25	31	11.3	16.3	10	35
3	Resumes Taleghani Hospital	16	12.5	28	16	14	10	10	5	12.5
4	Ibn sina & hijazi hospital	19	20	20	21	19	10	6.25	5	12.5
5	Dr. Sheikh Hospital	33	23.8	29	34	32	10	6.25	10	15
6	Mhvlat health network	31	43.8	33	31	29	10	16.3	5	22.5
7	Bardaskan network	26	27.5	28	27	30	13.8	10	5	22.5
8	Vlysrbrdskn hospital	28	28.8	31	31	33	8.75	8.75	5	22.5
9	Musa bin Jaafar Hospital Quchan	77	41.3	34	65.8	57.6	10	10	5	22.5
10	Shohada Hospital Quchan	29	25	30	23	29	10	15	5	22.5
11	Imam Khomeini Hospital Dargaz	20	18.8	23	20	21	6.25	11.3	15	22.5
12	Health network Dargaz	52.8	24.3	42.6	44.4	22.6	15	10	5	25
13	Sajjadiye hospital torbat jam	16	16.25	17	20	18	5	11.25	15	15
14	Torbat jam network	24	12.5	26	19	15	5	5	5	22.5
15	Hospital Khatamolanbiya Taybad	19	20	20	21	19	10	6.25	5	12.5
16	22 Bahman Hospital Khaf	37	28.8	33	33	31	11.3	10	5	22.5
17	Khaf city center	39	33.6	36	48	43	13.8	13.8	15	22.5
18	Hospital samen al chenaran	24	18.8	24	16	24	15	8.75	5	15
19	Chenaran network	18	15	18	22	26	5	12.5	7.5	40
20	Imam Khomeini Hospital Freeman	25	17.5	26	22	20	21.3	15	7.5	22.5
21	Freeman network	20	21.3	22	19	27	15	10	12.5	32.5
22	Samen Health Center	32	21.3	28	25	36	21.3	15	15	32.5
23	Mashhad health center (1)	19	21.3	33	18	28	23.8	15	20	27.5
24	Sarakhs Loghman Hospital	29	30	32	33	30	25	12.5	7.5	12.5
25	Sarakhs network	21	35	31	27	37	13.8	15	10	22.5
26	Omolbanin women's hospital	28	26.3	29	30	29	12.5	18.8	10	32.5
27	Omid hospital	41	41.3	42	43	43	21.3	15	10	30
28	Khatamolanbiya Eye Hospital	33	28.8	29	41	31	10	15	5	22.5
29	Health center no. 2, Mashhad	40	30	40	36	33	10	10	10	22.5
30	Mashhad health center (3)	20	27.5	31	22	28	6.25	11.3	5	22.5
31	Torghabe network and shandiz	31	30	34	31	35	6.25	11.3	10	22.5
32	College of dentistry	26	27.5	26	29	32	11.3	8.75	10	10
33	College of nursing and midwifery	22	33.8	35	39	46	15	10	5	25
34	Educational assistance	40	60	51	32	48	31.3	22.5	10	22.5
35	Cultural and student affairs	31	25	24	34	50	21.3	13.8	5	25
36	Food and drug administration	20	22.5	18	31	27	10	11.3	7.5	22.5
37	Assistance with health	35	61.3	41	44	42	7.5	10	5	25
38	College of paramedical sciences	53	27.5	35	29	29	10	13.8	5	30
39	College of health	25	45	29	35	32	2.5	2.5	5	12.5

Table 2. The inputs and the outputs of the Robust data envelopment analysis model.

Variable	MAX	MIN	MEAN
Input1	77	16	29.6872
Input2	61.25	12.5	28.2788
Input3	51	17	30.2974
Input4	65.8	16	30.2872
Input5	57.6	14	31.0821
Output1	31.25	2.5	12.3397
Output2	22.5	2.5	11.6667
Output3	20	5	8.01282
Output4	40	10	22.5641

Table 2 shows the inputs and the outputs of the RDEA model that we used for considering uncertainty in data. In this paper, we considered the uncertainty in output data and, by using *Models (1)-(8),* evaluated and ranked the health centers and hospitals.

Table 3. Evaluating and Ranking based on the CCR model.

	Health Centers	DEA-CCR	A and P	RDEA e=0.001	RDEA e=0.01	RDEA e=0. 1	RANK DEA-CCR	RANK RDEA
1	Ghaem research and treatment center	0.499389	-	0.4979940	0.485569	0.372501	35	34
2	Hashemi Nejad Hospital	0.949542	-	0.947445	0.928760	0.759992	14	13
3	Resumes Taleghani Hospital	1	1.001927	0.996758	0.967874	0.705022	11	10
4	Ibn sina& hijazi hospital	0.636565	-		0.612298	0.419213	28	27
5	Dr. Sheikh Hospital	0.535565	-	0.533894	0.518996	0.383429	33	32
6	Mhvlat health network	0.819428	-	0.817479	0.800115	0.642106	20	19
7	Bardaskan network	0.675388	-	0.673409	0.655766	0.500990	25	24
8	Vlysrbrdskn hospital	0.505327	-		0.491883	0.381895	34	33
9	Musa bin Jaafar Hospital Quchan	0.470682	-	0.469083	0.454834	0.326133	36	35
10	Shohada Hospital Quchan	0.918149	-	0.915856	0.895423	0.709706	16	15
11	Imam Khomeini Hospital Dargaz	1	1.043593	0.997733	0.977532	0.793699	8	7
12	Health network Dargaz	0.865652	-	0.863639	0.845697	0.682431	18	17
13	Sajjadiye hospital torbat jam	1	1.352941	0.997669	0.976898	0.787879	3	8
14	Torbat jam network	1	1.027210	0.997297	0.973212	0.754034	10	9
15	Hospital Khatamolanbiya Taybad	0.636565	-	0.634117	0.612298	0.419213	28	27
16	22 Bahman Hospital Khaf	0.550914	-	0.549520	0.537102	0.414098	32	31
17	Khaf city center	0.650891	-	0.649179	0.633924	0.500290	27	26
18	Hospital samen al chenaran	0.821586	-	0.819053	0.796478	0.591044	19	18
19	Chenaran network	1	1.708285	0.998002	0.980198	0.818182	1	1
20	Imam Khomeini Hospital Freeman	1	1.317282	0.997774	0.977938	0.797426	4	5
21	Freeman network	1	1.106108	0.997946	0.979648	0.813131	5	3
22	Samen Health Center	1	1.067107	0.997979	0.979973	0.816117	7	2
23	Mashhad health center (1)	1	1.686241	0.998002	0.980198	0.818182	2	1
24	Sarakhs Loghman Hospital	0.970276	-	0.968093	0.948642	0.771634	13	12
25	Sarakhs network	0.944700	-	0.942341	0.921317	0.729996	15	14
26	Omolbanin women's hospital	1	1.030199	0.997923	0.979418	0.811022	9	4
27	Omid hospital	0.656480	-	0.654997	0.641776	0.521469	26	25
28	Khatamolanbiya Eye Hospital	0.794164	-	0.792061	0.773317	0.606736	21	20
29	Health center no. 2, Mashhad	0.565538	-	0.564126	0.551538	0.436991	30	29
30	Mashhad health center (3)	0.745767	-	0.743671	0.724994	0.555028	33	22
31	Torghabe network and shandiz	0.564092	-	0.562488	0.548332	0.430520	31	30
32	College of dentistry	0.635688	-	0.633785	0.616827	0.426551	29	28
33	College of nursing and midwifery	0.686980	-	0.685375	0.671073	0.540919	24	23
34	Educational assistance	0.904206	-	0.902399	0.886301	0.739805	17	16
35	Cultural and student affairs	1	1.104767	0.997736	0.977560	0.793962	6	6
36	Food and Drug Administration	0.993218	-	0.990090	0.962217	0.708571	12	11
37	Assistance with health	0.428816	-	0.427561	0.416371	0.326458	37	36
38	College of paramedical sciences	0.775335	-	0.773632	0.758455	0.620350	22	21
39	College of health	0.305876		0.304689	0.294115	0.199722	38	37

In the Column 2 of *Table 3* by using the CCR model of DEA we evaluated and ranked the health centers and hospitals that shows centers 19, 23, 13,13, 20, 21, 35, 11, 26, 14, 35 are efficient DMUs and their degree of efficiency are one and then by using the A and P technique [15] respectively have the highest degree of efficiency. The obtained results by using the RDEA for controlling the uncertainty in output data show that the ranking of the efficient DMUs changed and led to a reduction in the degree of efficiency of DMUs. In general, using the RDEA reduces the degree of efficiency of DMUs, and by increasing the perturbation level (e) from 0.001 to 0.1, the degree of efficiency of DMUs is reduced. For example, Imam Khomeini Hospital Dargaz's degree of efficiency is 1 in the crisp model of CCR, but in our Robust model, the degree of efficiency is reduced to about 0.793699 when the level of uncertainty for all output parameters is 0.1.

Table 4. Evaluating and Ranking based on the BCC model.

	Health Centers	BCC	A and P	RDEA e=0.001	RDEA e=0.01	RDEA e=0.1	RANK DEA	RANK RDEA
1	Ghaem Research and Treatment Center	0.541439	-	0.540728	0.544393	0.502836	36	36
2	Hashemi Nejad Hospital	0.949542	-	0.947445	0.928760	0.774085	16	16
3	Resumes Taleghani Hospital	1	1.001927	1	1	1	11	4
4	Ibn Sina & Hijazi Hospital	0.987105	-	0.986031	0.977743	0.931609	14	14
5	Dr. Sheikh Hospital	0.688332	-	0.687805	0.683110	0.650096	28	28
6	Mhvlat Health Network	0.819428	-	0.817479	0.800115	0.658959	22	22
7	Bardaskan Network	0.771319	-	0.770492	0.763121	0.708976	26	26
8	Vlysrbrdskn Hospital	0.636373	-	0.635923	0.6	0.617951	33	33
9	Musa bin Jaafar Hospital Quchan	0.533730	-	0.533214	0.528611	0.509139	37	37
10	Shohada Hospital Quchan	0.918149	-	0.916372	0.900532	0.792017	18	18
11	Imam Khomeini Hospital Dargaz	1	1.043593	0.999064	0.990722	0.929111	8	10
12	Health Network Dargaz	0.865652	-	0.863639	0.845697	0.721932	20	20
13	Sajjadiye hospital torbat jam	1	1.352941	1	1	1	3	2
14	Torbat Jam Network	1	1.027210	1	1	1	10	3
15	Hospital Khatamolanbiya Taybad	0.987105	-	0.986031	0.977743	0.931609	14	14
16	22 Bahman Hospital Khaf	0.642360	-	0.641798	0.636789	0.599157	31	31
17	Khaf City Center	0.650891	-	0.649179	0.633924	0.535797	30	30
18	Hospital Samen al Chenaran	1	0.821586	1	1	1	13	5
19	Chenaran Network	1	1.708285	1	1	1	1	1
20	Imam Khomeini Hospital Freeman	1	1. 317282	0.999388	0.993938	0.944339	4	9
21	Freeman Network	1	1.106108	0.999798	0.998000	0.981637	5	6
22	Samen Health Center	1	1.066107	0.998689	0.987005	0.880685	7	12
23	Mashhad health center (1)	1	1.686241	0.999704	0.997066	0.973064	2	7
24	Sarakhs Loghman Hospital	0.970276	-	0968093	0.948642	0.799164	15	15
25	Sarakhs Network	0.944700	-	0.942341	0.921317	0.793766	17	17
26	Omolbanin Women's Hospital	1	1.030199	0.997990	0.980081	0.817106	9	13
27	Omid Hospital	0.656480	-	0.654997	0.641776	0.544235	29	29
28	Khatamolanbiya Eye Hospital	0.796016	-	0.793969	0.775733	0.614448	24	24
29	Health Center No. 2, Mashhad	0.580065	-	0.578899	0.568510	0.530044	35	35
30	Mashhad Health Center (3)	0.842091	-	0.841315	0.834399	0.813207	21	21
31	Torghabe Network and Shandiz	0.619495	-	0.618923	0.614141	0.597351	34	34
32	College of Dentistry	0.7624485	-	0.761713	0.754837	0.698081	27	27
33	College of Nursing and Midwifery	0.806781	-	0.806336	0.802370	0.766283	23	23
34	Educational Assistance	0.904206	-	0.902399	0.886301	0.739805	19	19
35	Cultural and Student Affairs	1	1.104167	0.998989	0.989981	0.908007	6	11
36	Food and Drug Administration	1	0.993218	0.999579	0.995829	0.968707	12	8
37	Assistance with Health	0.496361	-	0.495844	0.491238	0.467928	38	38
38	College of Paramedical Sciences	0.776108	-	0.774655	0.761703	0.661018	25	25
39	College of Health	0.64000	-	0.64000	0.64000	0.64000	32	32

In the Column 2 of *Table 4* by using the BCC model of DEA we evaluated and ranked the health centers and hospitals that shows centers 19, 23, 13, 20, 21, 35, 22, 11, 26, 14, 3, 36, 18 are efficient DMUs and their degree of efficiency are one and then by using the A and P technique [15] respectively have the highest degree of efficiency. The obtained results by using the RDEA for controlling the uncertainty in output data show that the rank of the efficient DMUs changed and led to a reduction in the degree of efficiency of DMUs. In general, by using the RDEA, the degree of efficiency of DMUs decreased, and by increasing the perturbation level (e) from 0.001 to 0.1, the degree of efficiency of DMUs decreased. For example, Imam Khomeini Hospital Dargaz's degree of efficiency is 1 in the crisp model of BCC, but in our Robust model, the degree of efficiency is reduced to about 0.929111 when the level of uncertainty for all output parameters is 0.1. This fact shows that the precision of estimated data has a significant influence on the results of efficiency measurement. By using statistical tests based on Spearman Pearson [16] for comparing the obtained results from the DEA and RDEA ranking, as follows:

Where n is the number of units and d_i is calculated as the difference between the rankings of DEA and Robust DEA [6].

$$d_{spearman}=1-\frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}.$$

By using the results of *Table 3*, $d_{spearman}$= 0.976518, which means, statistically, there is no significant difference between the DEA and RDEA ranking in the CCR model of DEA.

By using the results of *Table 4*, $d_{spearman}$=0.969636, which means, statistically, there is no significant difference between the DEA and RDEA ranking in the BCC model of DEA.

By using the results of *Table 3* and *Table 4*, $d_{spearman}$=0.924899, which means, statistically, there is no significant difference between the ranking of CCR and the BCC model of DEA.

By using the results of *Table 3* and *Table 4*, $d_{spearman}$=0.8848817, which means, statistically, there is no significant difference between the ranking of RDEA based on the CCR and BCC model.

4 | Conclusion

Considering the results obtained with this setup, which reached, though there is a difference in ranking DMUs between CRS and the VRS approaches, there is no significant difference between the ranking of RDEA based on the CCR and BCC models, and there is no significant difference between the ranking of the BCC and CCR models of DEA. Results show that considering the uncertainty in the data can lead to a loss of efficiency scores. And by increasing the perturbation level (e) from 0.001 to 0.1, the efficiency of DMUs was reduced.

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