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Efficiency Analysis of Tehran Hospitals' Emergency Departments through Data Envelopment Analysis with Undesirable Factors

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Abstract

In this paper, the efficiency of Tehran hospital emergency departments is measured using Data Envelopment Analysis (DEA) when undesirable input and output factors are present. As traditional DEA models cannot directly handle undesirable factors, this research has overcome this limitation through a redefined framework. The proposed approach first determines the production possibility set under the given problem assumptions. Then it develops a new DEA model to evaluate Decision Making Unit (DMU) efficiency, taking into account the impact of undesirable factors on the efficiency frontier. The utilization of the model with real data from Tehran Hospital's emergency departments confirms the need to account for undesirable factors when measuring efficiency and developing improvement plans. This research helps measure emergency department efficiency and manage unwanted factors.

Keywords: Data envelopment analysis, Undesirable inputs and outputs, Efficiency, Efficient frontier.

1 | Introduction

Data Envelopment Analysis (DEA), a recognized method for efficiency evaluation, has its roots in the notion of Pareto non-dominated units and was developed by Koopman [1] and Farrell [2] in the fields of production and efficiency appraisal [3]. Charnes et al. [4] employed linear programming to propose an approach for identifying efficient frontiers and evaluating productivity, encompassing both input-oriented and output-oriented models. DEA has been confirmed as a robust methodology in performance evaluation [5].

The initial DEA models focused on enhancing efficiency by utilizing specified inputs and outputs; however, Koopman [1] also introduced the concept of unwanted outputs. In numerous practical problems, some inputs

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and outputs may have an inverse influence on efficiency; for example, reducing unwanted inputs in recycling or decreasing unwanted outputs, such as waste and deaths, can result in enhanced efficiency.

Various methods exist for incorporating undesirable outputs in DEA, which are divided into two classes: Direct and indirect. Indirect methods transform unwanted inputs and outputs into desirable values using monotonic functions [1], [6], [7]. Direct methods involve assumptions about the production possibility set [2].

Efficiency evaluation in the presence of unwanted outputs has received attention since the 1980s [8], [9]. Various researchers have presented various approaches for estimating shadow prices of undesirable outputs [10] and assessing productivity utilizing directional distance functions [11]. Additionally, Slacks-Based Measure (SBM) models [12] and various approaches have been developed to take into account the type of data in DMUs [13]–[22].

DEA models typically consider units under assessment as black boxes; however, recent research has shown that ignoring internal structure and unwanted factors can lead to incorrect efficiency evaluations. Kao [23] demonstrated through a model that aligning inputs and outputs of inefficient units with DEA results is possible.

In reviewing the literature, it has been identified that the common efficient frontier of Decision Making Units (DMUs) may not be unique. Young et al. [24] and Zho et al. [25] have addressed this topic and suggested methods for achieving a unique efficient frontier. Furthermore, models have been developed to address the issue of one-dimensional output [26] and to account for undesirable outputs with fixed values [27], [28].

Kao and Hwang [29] proposed a model for assessing efficiency in the presence of unwanted outputs with shadow price changes. Wang et al. [30] employed a meta-frontier method to compare the efficiency of carbon reduction technology systems. Dakpo et al. [31] and Pham and Zelenyuk [32] have presented a critical review of technology models with unwanted outputs. Podinovski [33] has addressed the estimation of marginal indices of nonparametric production frontiers in the presence of undesirable outputs.

Cross-evaluation methods of productivity have also been suggested as an option for ranking DMUs. Shi et al. [34] have presented a new method of cross-efficiency evaluation in which each DMU has an impartial attitude toward other similar units.

To address the issue of assessment depending on various frontiers, Young et al. [24] were the first to propose a DEA model with a common frontier for efficiency evaluation in the presence of unwanted outputs. Their model assesses the efficiency of DMUs in an environment with unwanted outputs, employing a nonparametric linear programming approach to establish a common frontier.

In this research, the production possibility set is presented in accordance with the notion of unwanted inputs and outputs, and the efficiency of DMUs is investigated as a real-world instance with the presence of unwanted inputs and outputs.

The primary purpose of the current research is to assess the efficiency of emergency departments in Tehran hospitals by employing a suitable DEA model that can simultaneously consider both wanted and unwanted outputs. Considering the significance of emergency department performance in delivering healthcare services, the current study can help identify the strengths and weaknesses of these departments and provide insights into improving their performance.

To achieve this objective, the study develops and designs a DEA model that integrates unwanted outputs into the efficiency assessment process. This model, which examines the typical features of hospital emergency departments, can help identify factors influencing their efficiency and deliver solutions for enhancing performance.

In this respect, the primary study questions are:

I. How can an appropriate DEA model be created for evaluating the efficiency of hospital emergency departments while considering undesirable outputs?

II. What factors have the most significant influence on the efficiency of emergency departments in Tehran hospitals?

III. What solutions can be offered to enhance the efficiency of emergency departments in Tehran hospitals?

To address these questions, an appropriate DEA model will be developed to evaluate the effectiveness of hospital emergency departments, taking into account undesirable outputs. Then, using data from emergency departments in Tehran hospitals, the efficiency of these departments will be assessed. Ultimately, by analyzing the outcomes of the effectiveness assessment, factors that influence the efficiency of emergency departments will be identified, and solutions for enhancing their performance will be proposed.

2 | Research Methodology

The current study, which aims to present a novel model in the field of effectiveness assessment, falls within the category of basic developmental research in terms of its objective. The adopted research approach is based on library studies and quantitative data analysis. Regarding data gathering, the research employs a descriptive-analytical method, focusing on a case study.

2.1 | Data Collection

The data needed for the research is gathered in two parts:

Library studies: In this section, using note-taking tools, credible scientific sources, such as books, papers, and other research published in international databases (e.g., Springer and Elsevier), as well as relevant domestic sources, are studied. These investigations create the theoretical basis and conceptual framework of the study.

Field data: After creating the presented model and executing it using GAMS software, a numerical example will be introduced to illustrate the model's application. Additionally, the actual data needed for the case study are gathered through field collection from hospitals in Tehran province, with consideration for ethical principles and respect for the privacy of patients and staff.

3 | Production Possibility Set

A function that maps a set of inputs to a subset of outputs, such that these inputs can produce those outputs.

$$L_{(y)} = \{x \mid (x, y) \in T\}.$$

We also define the set of outputs $P_{(x)}$ corresponding to input x as follows:

$$P(x) = \{y \mid (x, y) \in T\}.$$

In this research, we have considered inputs as ordered pairs $x = (x^D, x^I)$ where $x^D = (x_1^D, \ldots, x_{m_1}^D)$ and $x^I = (x_1^I, \ldots, x_{m_2}^I)$ are desirable and undesirable inputs, in turn. Additionally, $y = (y^g, y^b)$ represents the outputs where $y^g = (y_1^g, \ldots, y_{s_1}^g)$ are desirable outputs and $y^b = (y_1^b, \ldots, y_{s_2}^b)$ stands for undesirable ones.

Definition 1. DMU (x^D, x^I, y^g, y^b) is said to dominate unit (x'^D, x'^I, y'^g, y'^b) whenever $x^D \leq x'^D$, $x^I \geq x'^I$, $y^g \geq y'^g$ and $y^b \leq y'^b$ exists with at least one strict inequality. That is:

$$\begin{pmatrix} -x^D \\ x^I \\ y^g \\ -y^b \end{pmatrix} \geq \begin{pmatrix} -x'^D \\ x'^I \\ y'^g \\ -y'^b \end{pmatrix}.$$

A DMU is efficient if no unit in T dominates it. We consider the production possibility set T with the following characteristics:

I. T is convex.

II. T is closed.

III. Monotonicity property for inputs and desirable outputs, meaning:

$$\text{for all } u \in R_{+}^{m_1}, v \in R_{+}^{s_1}, (x^D, x^I, y^g, y^b) \in T \Rightarrow (x^D + u, x^I, y^g - v, y^b) \in T.$$

This property cannot necessarily exist for inputs and undesirable outputs, because in that case, T would lack efficient units. Taking the above conditions into account, we define the set T as follows:

$$T = \left\{ (x^D, x^I, y^b, y^g) \left| \begin{array}{c} x^D \geq \sum_{j=1}^{n} \lambda_j x_j^D, x^I = \sum_{j=1}^{n} \lambda_j x_j^g, y^b = \sum_{j=}^{n} \lambda_j y_j^b, y^g \leq \sum_{j=1}^{n} \lambda_j y_j^g \\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \ldots, n \end{array} \right. \right\}.$$

4 | Determining Efficiency

To assess the efficiency of the unit under examination in the given context, we aim to enhance it by reducing desirable inputs and increasing undesirable inputs. In the output context, we improve the unit under evaluation by increasing desirable outputs and decreasing undesirable outputs. Considering this, a model was suggested by Farrell [2] to enhance preferable outcomes and reduce unfavorable outputs; however, the problem with this model was its non-linearity.

In the [TRβ] method, while improving desirable outputs, it simultaneously decreases undesirable outputs. However, the problem with this method is that the efficiency measure depends on the β value, and as β increases, the efficiency number becomes larger for inefficient DMUs.

Some other methods, such as [WD] and [MLT], operate in such a way that reducing undesirable outputs is just achievable with a reduction in desirable outputs. However, we believe that efficiency advancement occurs when desirable outputs increase or undesirable outputs decrease, which we will examine in both input-oriented and output-oriented contexts.

4.1 | Input Context

Suppose $\mathrm{DMU_o} = (x_o^D, x_o^I, y_o^g, y_o^b)$ is the unit under evaluation. Corresponding to output $y_o = (y_o^g, y_o^b)$ for the set of inputs $L(y_o^g, y_o^b)$, according to the definition we have:

$$L(y_o^g, y_o^b) = \left\{ (x^D, x^I) \middle| (x^D, x^I, y_o^g, y_o^b) \in T \right\}.$$

We consider a subset of $L(y_o^g, y_o^b)$ in the following manner:

$$\partial^p L(y_o^g, y_o^b) = \left\{ (x^D, x^I) \middle| \forall (u,v) \geq 0, (u,v) \neq 0 \quad \text{then} \quad (x^D - u, x^I + v) \notin L(y_o^g, y_o^b) \right\}.$$

$\partial L(y_o^g, y_o^b)$ includes the inputs corresponding to all efficient DMUs that can produce output (y_o^g, y_o^b).

In determining the efficiency of $\mathrm{DMU_o}$, we are simultaneously seeking the maximum reduction in x_o^D and the maximum increase in x_o^I, in order to push the unit under evaluation toward the efficient frontier of $\partial L(y_o^g, y_o^b)$. That is:

$$NE^d(x_o, y_o) = \sup\left\{ \theta \middle| \; x_o - \theta d_o \in L(y_o) \right\}.$$

$NE^d(x_o, y_o)$ is the measure of inefficiency of the unit under evaluation. $d_o = ({d_o}^D, {d_o}^I)$ determines the direction of movement for the unit under evaluation toward the frontier. ${d_o}^D \in R_+^{m_1}$ and ${d_o}^I \in R_-^{m_2}$ ensure that this movement leads to a reduction in desirable inputs and an increase in undesirable inputs. In this research, we direct the desirable input radially toward the efficient frontier; therefore:

$${d_o}^D = {x_o}^D.$$

And we consider the direction of increasing the undesirable input as follows:

$${d_o}^I = {x_o}^I - x^I_{max}.$$

such that:

$$(x^I_{max})_i = Max_j \left\{x^I_{ij}\right\}.$$

Hence, taking into account the definition of inefficiency, we have:

$$\begin{aligned}
&\theta_o^* = Max \quad \theta_o \\
&s.t. \\
&\quad \sum_{j=1}^{n} \lambda_j x_j^D + s^- = x_o^D - \theta_o {d_o}^D, \\
&\quad \sum_{j=1}^{n} \lambda_j x_j^I = x_o^I - \theta {d_o}^I, \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^g - s^+ = y_o^g, \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^b = y_o^b, \\
&\quad \sum_{j=1}^{n} \lambda_j = 1, \\
&\quad \lambda_j \geq 0, \quad \text{for all} \quad j = 1, \ldots, n.
\end{aligned} \tag{1}$$

Based on the definition of the production possibility set of T that we defined in the previous section, *Model (1)* is feasible in this set.

Theorem 1. The unit under evaluation in *Model (1)* is efficient if and only if:

I. $\theta_0^* = 1.$

II. The slack variables in all optimal solutions are zero.

Theorem 2. If θ^* is the optimal solution of *Model (1)* in evaluating DMU_o, then:

$$(x^D - \theta^* d^D - s^{-^*}, x^I - \theta^* d^I) \in \partial^p L(y_o^b, y_o^g).$$

s^{-^*} is one of the optimal solutions.

4.2 | Nature of Output

Let's assume $DMU_o = (x_o^D, x_o^I, y_o^g, y_o^b)$ is the unit under evaluation. Corresponding to the input $x_o = (x_o^D, x_o^I)$ for the set of outputs $p(x_o^D, x_o^I)$ according to the definition we have:

$$p(x_o^D, x_o^I) = \left\{(y^g, y^b) \middle| (x_o^D, x_o^I, y^g, y^b) \in T\right\}.$$

And we consider a subset of $p(x_o^D, x_o^I)$ as follows:

$$\partial^p p(x_o^D, x_o^I) = \left\{ (y^g, y^b) \middle| \text{for all } (u,v) \geq 0, (u,v) \neq 0 \quad \text{then} \quad (y^g + u, y^b - v) \notin p(x_o^D, x_o^I) \right\}.$$

In determining the efficiency of DMU_o in terms of output, we are simultaneously seeking the maximum increase in y_o^g and the maximum decrease in y_o^b, in order to push the unit under evaluation toward the efficient frontier of $\partial^p p(x_o^D, x_o^I)$. That is:

$$NE^d(x_o, y_o) = \sup\left\{ \beta \middle| y_o + \beta d \in p(x_o) \right\}.$$

$d = (d^g, d^b)$ determines the direction of movement for the unit under evaluation toward the frontier such that $d^g \in R_+^{s_1}$ and $d^b \in R_-^{s_2}$ lead to an increase in desirable outputs and a decrease in undesirable outputs.

In this research, we direct the desirable outputs radially toward the efficient frontier; therefore:

$d^g = y_o^g$.

And similarly, we reduce the undesirable outputs in a radial direction, meaning:

$d^I = -y_o^b$.

Therefore, according to the definition, we have:

$$\begin{aligned}
\beta_o^* = \text{Max} \quad & \beta_o \\
\text{s.t.} \quad & \\
& \sum_{j=1}^{n} \lambda_j x_j^D + s^- = x_o^D, \\
& \sum_{j=1}^{n} \lambda_J x_j^I = x_o^I, \\
& \sum_{j=1}^{n} \lambda_j y_j^g - s^+ = y_o^g + \beta_o d_o^g, \\
& \sum_{j=1}^{n} \lambda_j y_j^b = y_o^b + \beta_o d_o^b, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& \lambda_j \geq 0 \text{ for all } j = 1, \ldots, n.
\end{aligned} \tag{2}$$

Theorem 3. A unit under evaluation in *Model (2)* is efficient if and only if:

I. $\beta_o^* = 1$.

II. The slack variables are zero in all optimal solutions.

Theorem 4. If β_o^* is the optimal solution of *Model (2)* in evaluating DMU_o, then:

$$(y_o^* + \beta_o^* d_o^g + s^{+^*}, y_o^b + \beta_o^* d_o^b) \in \partial^p p(x_o^D, x_o^I).$$

s^{+^*} is one of the optimal solutions.

4.3 | Hospital Emergency Department Case Studies Employing the Presented Models

Medical treatment is a complex system that encompasses primary and secondary treatment, as well as subsequent care stages. At the center of this complex system are hospitals. Among hospital departments, emergency rooms stand out as a central part of the medical treatment system, with their 24-hour operations and high patient volume. Emergency departments operate 24 hours a day, 365 days a year, providing care to patients requiring emergency, semi-emergency, and non-emergency medical attention.

On a national scale, emergency departments admit and treat more than 33 million patients annually, making them critical components of the healthcare infrastructure. Yet, the pervasive problem of severe overcrowding is a global issue that hospitals must grapple with. The ever-increasing number of patients, along with other challenges facing healthcare systems, puts pressure on the scarce resources of hospitals. Timely and acceptable emergency care in such situations becomes a significant issue, which may result in delays in providing the required medical care to patients.

One of the fundamental tasks for healthcare managers is the effective allocation of human resources and assets without compromising the quality of care. In the current research, we employed DEA models to evaluate the effectiveness of hospital emergency departments and identify opportunities for improvement. Conventional DEA models typically do not manage the decline of unwanted outputs. Hence, this study incorporated a suggested model to evaluate the effectiveness of DMUs in the company, considering both desirable and undesirable inputs and outputs. For the presented research, data were gathered from emergency departments of 30 Tehran hospitals, concentrating on five desirable inputs, one unwanted input, four desirable outputs, and one unwanted output, with the subsequent indicators:

Data indicators

Number of incoming patients: Total number of patients visiting the emergency department during a specified time period.

Patient triage level: Classification of patients based on the severity of their condition (levels 1 to 5).

Number of treated patients: Number of patients discharged from the emergency department after receiving medical services.

Time spent in the emergency department: Average duration patients spend in the emergency room, categorized as:

I. Less than 12 hours

II. More than 12 hours

Staying time for services: The Duration patients stay to receive medical treatment.

Referral rate to specialized departments: Percentage of patients referred to specialized departments.

Staff-to-patient ratio: Number of nurses and doctors relative to the number of incoming patients.

Available equipment and facilities: Number of hospital beds and medical equipment obtainable.

The data, depending on the mentioned indicators, is presented in *Table 1* as follows:

Table 1. Data collected from hospital emergency departments with desirable and unwanted inputs and outputs.

DMU	Inputs					Unwanted Inputs	Unwanted Outputs	Outputs			
DMUs	X_1	X_2	X_3	X_4	X_5	X_b	y_b	Y_1	Y_2	Y_3	y_4
DMU_1	18	1	1	38	27	19	2	1155	295	265	32
DMU_2	19	2	1	41	15	4	3	1254	338	305	30
DMU_3	21	2	2	42	17	9	1	1259	325	261	28
DMU_4	19	2	1	39	21	14	4	1244	320	263	29
DMU_5	20	2	1	40	25	17	2	1254	323	271	29
DMU_6	22	2	2	42	34	7	7	917	125	169	22
DMU_7	21	2	1	41	26	15	3	1245	332	237	28
DMU_8	21	2	1	41	18	11	2	1254	323	270	28
DMU_9	20	2	1	40	19	8	1	1204	340	265	27

Table 1. Continued.

DMU	Inputs					Unwanted Inputs	Unwanted Outputs	Outputs			
DMUs	X_1	X_2	X_3	X_4	X_5	X_b	y_b	Y_1	Y_2	Y_3	y_4
DMU_{10}	20	2	1	39	17	6	1	1254	315	270	29
DMU_{11}	20	2	1	39	18	10	2	1260	324	272	29
DMU_{12}	19	1	2	39	29	17	4	944	192	246	30
DMU_{13}	18	1	2	38	29	9	5	985	194	240	28
DMU_{14}	19	1	2	40	30	13	1	1085	295	226	32
DMU_{15}	19	1	2	39	30	12	7	764	162	116	20
DMU_{16}	20	1	2	41	31	5	5	691	150	244	19
DMU_{17}	20	1	2	42	31	8	3	994	192	246	28
DMU_{18}	20	1	2	41	25	9	4	931	201	256	29
DMU_{19}	21	1	2	42	26	18	2	941	188	274	28
DMU_{20}	20	1	2	41	25	16	1	1145	284	275	27
DMU_{21}	21	1	2	41	25	19	5	948	193	212	32
DMU_{22}	18	1	1	38	27	6	4	994	305	266	28
DMU_{23}	20	1	2	39	26	5	2	941	245	246	27
DMU_{24}	19	1	2	39	27	15	3	984	189	274	29
DMU_{25}	20	1	2	41	27	7	2	948	193	247	28
DMU_{26}	22	2	2	43	14	11	1	1259	335	271	30
DMU_{27}	23	2	2	44	32	5	6	1015	224	261	24
DMU_{28}	21	2	1	42	13	13	1	1370	365	322	35
DMU_{29}	22	2	1	42	15	18	2	1244	320	270	29
DMU_{30}	23	2	2	44	14	8	5	1154	314	272	31

Table 2. Statistical information.

Inputs	Minimum	Maximum	Mean	Standard Deviation
E.R. Personnel	20	27	23.17	1.79
Beds available	38	44	40.60	1.67
Waiting time	13	34	23.77	6.21
Deceased patients	12	84	36.40	21.94
Persons discharged	8292	16440	13046.82	2040.42
Hospitalization less than 12 hours	3336	8244	6205.20	1184.44
Hospitalization over 12 hours	228	420	338	40.12

Example 1. We consider 30 DMUs from the data of Tehran hospitals, as outlined in *Table 1*. We evaluated this data using the BCC model and the suggested input-oriented model with undesirable factors. The results of this evaluation, along with the presented model, are as follows.

Table 3. Results obtained using the presented model 1 (Undesirable input).

DMUs	θ_o^*	BCC	DMUs	θ_o^*	BCC	DMUs	θ_o^*	BCC
DMU_1	1	1	DMU_{11}	0.9905	1	DMU_{21}	1	1
DMU_2	0.8952	1	DMU_{12}	1	0.9375	DMU_{22}	0.8859	1
DMU_3	0.9122	0.919	DMU_{13}	0.9163	0.9048	DMU_{23}	0.9663	1
DMU_4	0.9889	1	DMU_{14}	0.9963	1	DMU_{24}	0.9959	1
DMU_5	1	0.9711	DMU_{15}	0.7068	0.6615	DMU_{25}	0.8759	0.9169
DMU_6	0.7732	0.6693	DMU_{16}	0.7692	0.8873	DMU_{26}	0.9463	0.919
DMU_7	1	0.944	DMU_{17}	0.9039	0.9146	DMU_{27}	0.7901	0.8106
DMU_8	0.9477	0.9405	DMU_{18}	0.9898	1	DMU_{28}	0.9893	1
DMU_9	0.9763	1	DMU_{19}	1	0.9986	DMU_{29}	1	0.908
DMU_{10}	0.9689	1	DMU_{20}	1	1	DMU_{30}	0.7964	0.8857

In agreement with the results in *Table 3*, units with the highest unwanted inputs, such as DMU_5 and DMU_7, have experienced increased efficiency. Similarly, units like DMU_{20}, DMU_1, and DMU_{21} have held their effectiveness and reached the efficiency frontier. Interestingly, some units that employed fewer unwanted inputs compared to other units have switched from efficient to inefficient or have seen a decrease in efficiency scores. The reverse situation also holds true.

Accordingly, we deduce that as undesirable inputs increase in the units being evaluated and the system (To the extent allowed by the production possibility set noting that in hospitals, equipment cannot be sterilized more than two or three times, so the feasibility of the set remains intact, meaning we cannot excessively use undesirable inputs to the point of leaving the PPS), efficiency increases, and vice versa. These factors influence efficiency determination, and in accordance with the real-world hospital example, the suggested model is admissible, and these principles hold true within it.

Example 2. We evaluated 30 DMUs from the data of Tehran hospitals using both the BCC model and the proposed model. The results, along with the proposed model for unwanted output, are as follows.

Table 4. Results obtained from using the new proposed model (Undesirable outputs).

DMUs	β_o^*	BCC	DMUs	β_o^*	BCC	DMUs	β_o^*	BCC
DMU_1	1	1	DMU_{11}	0.9905	1	DMU_{21}	0.9663	1
DMU_2	0.9994	1	DMU_{12}	0.8457	0.9375	DMU_{22}	0.9859	1
DMU_3	1	0.919	DMU_{13}	0.7548	0.9048	DMU_{23}	1	1
DMU_4	0.8997	1	DMU_{14}	1	1	DMU_{24}	0.9959	1
DMU_5	0.969	0.9711	DMU_{15}	0.5568	0.6615	DMU_{25}	0.9759	0.9169
DMU_6	0.5782	0.6693	DMU_{16}	0.7895	0.8873	DMU_{26}	1	0.919
DMU_7	0.9108	0.944	DMU_{17}	0.9036	0.9146	DMU_{27}	0.7801	0.8106
DMU_8	0.9377	0.9405	DMU_{18}	0.9818	1	DMU_{28}	1	1
DMU_9	1	1	DMU_{19}	1	0.9986	DMU_{29}	0.9658	0.908
DMU_{10}	1	1	DMU_{20}	1	1	DMU_{30}	0.7324	0.8857

From the results in *Table 4*, efficiency for all units was calculated using the BCC and suggested models, and comparisons were made. These results indicate that the units with the highest undesirable outputs, specifically DMU_2 and DMU_4, have experienced a decline in efficiency and have even transitioned from efficient to inefficient. The converse condition also holds.

Furthermore, inefficient units, such as DMU_3, DMU_{19}, DMU_{23}, and DMU_{26} in the BCC output model with variable returns to scale, have become efficient since they have low mortality rates in these hospitals. It is therefore established that unwanted outputs must be included in the DMU evaluation process, as excluding them would result in the failure to achieve desired outcomes and, in some instances, lead to the classification of inefficient units as efficient ones.

5 | Conclusion

This research aimed to evaluate the effectiveness of DMUs in the presence of unwanted input and output factors, presenting several models. The application of these models in evaluating emergency departments of Tehran hospitals demonstrated that the presence of unwanted factors has a consequential influence on specifying the efficiency frontier and cannot be overlooked. The suggested models, by taking into account these factors, compare DMUs with corresponding units on the efficient frontier and enable the identification and implementation of efficiency advancement methods. These methods involve optimal management of inputs (Advancing desirable inputs and reducing undesirable inputs) and outputs (Increasing desirable outputs and reducing undesirable ones), which can enhance the efficiency of DMUs and bring them closer to the efficient frontier. In General, the results of this research highlight the significance of addressing undesirable factors in efficiency measurement and providing solutions for improving the performance of DMUs.

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