Annals of Healthcare Systems Engineering

www.ahse.reapress.com

Ann. Healthc. Syst. Eng. Vol. 2, No. 4 (2025) 239-249.

Paper Type: Original Article

Medical Knowledge Hypergraphs and Medical Knowledge Superhypergraphs

Takaaki Fujita*

Independent Researcher, Shinjuku, Shinjuku-ku, Tokyo, Japan; Takaaki.fujita060@gmail.com.

Citation:

Received: 19 February 2025 *Revised: 16 July 2025* *Accepted: 12 September 2025*	Fujita, T. (2025). Medical knowledge Hypergraphs and medical knowledge Superhy-pergraphs. *Annals of healthcare systems engineering*, 2(4), 239-249.

Abstract

A finite *hypergraph* extends the classical graph model by allowing *hyperedges* that may connect any nonempty subset of vertices [1, 2, 3]. Building on this foundation, a finite *SuperHyperGraph* is obtained by iteratively applying the powerset construction, thereby forming nested families of vertex and edge sets that encode multi-layered relationships [4, 5]. A *Knowledge Graph* is a graph-based representation that encodes facts as entities together with their relations, supporting reasoning, semantic search, and knowledge-driven applications. A *Medical Knowledge Graph* adapts this paradigm to the medical domain, modeling entities such as diseases, symptoms, drugs, and procedures, and linking them through clinically meaningful relations to facilitate decision support.

In this paper, we extend the Medical Knowledge Graph framework using HyperGraphs and SuperHyperGraphs, and investigate its properties. A *Medical Knowledge HyperGraph* further generalizes the framework by permitting hyperedges that simultaneously connect multiple medical entities, thereby capturing complex clinical relationships not representable with simple triples. Finally, a *Medical Knowledge SuperHyperGraph* introduces hierarchical layers via iterated powersets, enabling the representation of multi-level medical relations and providing a unifying model that encompasses graphs, hypergraphs, and typed medical knowledge structures.

Keywords: Superhypergraphs, Hypergraphs, Medical knowledge graphs, Knowledge graphs.

1|Introduction

This section collects the basic terminology and notation on which the rest of the paper relies. Unless explicitly stated, all graphs discussed below are *finite*.

Corresponding Author: Takaaki.fujita060@gmail.com

https://doi.org/10.22105/ahse.vi.37

1.1| SuperHyperGraphs

A finite *hypergraph* extends the classical graph model by allowing *hyperedges* that may join any nonempty subset of vertices [1, 2, 3]. Building on this idea, a finite *SuperHyperGraph* is obtained by iterating the powerset construction so as to form nested families of vertex and edge sets that encode multi-layer relations [4, 6, 7, 8, 5]. Such models have proved useful in, for example, molecular design, complex-network analysis, and advanced signal-processing workflows [9, 10]. Unless noted otherwise, the integer n in $\mathcal{P}_n(\cdot)$ or in an n–SuperHyperGraph is assumed to be nonnegative.

Definition 1.1 (Base Set)**.** A *base set* S is the underlying universe of discourse:

$$S \;=\; \{\, x \mid x \text{ belongs to the context under consideration} \,\}.$$

All objects appearing in $\mathcal{P}(S)$ or in any iterated powerset $\mathcal{P}_n(S)$ are, by definition, subsets ultimately drawn from S.

Definition 1.2 (Powerset)**.** (cf. [11, 12]) For a set S, the *powerset* $\mathcal{P}(S)$ is the family of all subsets of S:

$$\mathcal{P}(S) \;=\; \{\, A \subseteq S \,\}.$$

In particular, both S and the empty set $\emptyset$ are members of $\mathcal{P}(S)$.

Definition 1.3 (Hypergraph)**.** [13, 14] A *hypergraph* is an ordered pair $H = (V, E)$ where

- V is a finite set of vertices, and
- E is a finite collection of nonempty subsets of V; elements of E are the *hyperedges*.

Hypergraphs naturally capture interactions among more than two participants.

Example 1.4 (Hypergraph: Sepsis Initial Management Bundle)**.** Consider the finite vertex set

$$V = \{\text{BC, ABX, IVF, LAC, VP}\},$$

where BC = blood cultures, ABX = broad-spectrum antibiotic, IVF = intravenous fluids, LAC = serum lactate, VP = vasopressor. A hypergraph $H = (V, E)$ can encode clinical bundles via hyperedges that represent multi-way order sets:

$$E = \big\{\{\text{BC}, \text{ABX}, \text{LAC}, \text{IVF}\}, \{\text{IVF}, \text{VP}\}\big\}.$$

The first hyperedge models an initial "sepsis bundle" (obtain cultures, give antibiotics, draw lactate, start fluids) and the second models the joint requirement of fluids and vasopressor for persistent hypotension. By Definition 1.3, E is a finite family of nonempty subsets of V, hence H is a finite hypergraph.

Definition 1.5 (n-th Powerset)**.** [15, 16] Let X be a set. Define $\mathcal{P}_1(X) = \mathcal{P}(X)$. For $n \geq 1$,

$$\mathcal{P}_{n+1}(X) \;=\; \mathcal{P}\big(\mathcal{P}_n(X)\big).$$

When the empty set is excluded, we write $\mathcal{P}^*_n(X) = \mathcal{P}_n(X) \setminus \{\emptyset\}$.

Example 1.6 (n-th Powerset: Diagnostic Imaging Panels)**.** Let

$$X = \{\text{CT, MRI, US}\}$$

denote three imaging modalities (computed tomography, magnetic resonance imaging, ultrasound). The nonempty first powerset

$$\mathcal{P}_1^*(X) = \mathcal{P}(X) \setminus \{\emptyset\}$$

lists all admissible single-visit "panels" of modalities; explicitly,

$$\mathcal{P}_1^*(X) = \big\{\{\text{CT}\}, \{\text{MRI}\}, \{\text{US}\}, \{\text{CT}, \text{MRI}\}, \{\text{CT}, \text{US}\}, \{\text{MRI}, \text{US}\}, \{\text{CT}, \text{MRI}, \text{US}\}\big\},$$

so $|\mathcal{P}_1^*(X)| = 2^3 - 1 = 7$. Elements of the second powerset $\mathcal{P}_2(X) = \mathcal{P}\big(\mathcal{P}_1(X)\big)$ are collections of such panels; for instance,

$$T_1 = \big\{\{\text{CT}\}, \{\text{MRI}, \text{US}\}\big\}, \qquad T_2 = \big\{\{\text{CT}, \text{MRI}\}, \{\text{US}\}\big\}$$

are two distinct members of $\mathcal{P}_2^*(X)$. Here T_1 can represent two alternative workups (CT alone vs. MRI+US), while T_2 encodes a staged plan (CT+MRI followed by US). This illustrates how $\mathcal{P}_n$ organizes increasingly rich combinatorial choices of medical tests as n grows.

Definition 1.7 (n–SuperHyperGraph)**.** (cf. [17, 18, 19]) Fix a finite, nonempty base set V_0 and define the iterated powerset by

$$\mathcal{P}^0(V_0) \coloneqq V_0, \qquad \mathcal{P}^{\,k+1}(V_0) \coloneqq \mathcal{P}\big(\mathcal{P}^{\,k}(V_0)\big) \quad (k \in \mathbb{N}).$$

For an integer $n \geq 0$, an *n–SuperHyperGraph on V_0* is a pair

$$\text{SHG}^{(n)} = (V, E)$$

with

$$V \subseteq \mathcal{P}^{\,n}(V_0) \quad \text{and} \quad E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

Elements of V are the *n–supervertices*, and elements of E are the *n–superedges* (each n–superedge is a nonempty subset of V).

Example 1.8 (n–SuperHyperGraph (here $n = 2$): Two-Tier Care Pathway)**.** Let the base set of atomic actions be

$$V_0 = \{\text{ABX, BC, LAC, IVF, VP}\}.$$

Elements of $\mathcal{P}^2(V_0)$ are collections of nonempty subsets of V_0, which we use to represent stage-wise groupings of actions. Define three 2–supervertices

$$A_1 = \big\{\{\text{ABX}, \text{BC}\}, \{\text{LAC}\}\big\}, \quad A_2 = \big\{\{\text{IVF}\}, \{\text{VP}\}\big\}, \quad A_3 = \big\{\{\text{ABX}\}, \{\text{IVF}, \text{VP}\}\big\},$$

and set $V = \{A_1, A_2, A_3\} \subseteq \mathcal{P}^2(V_0)$. Form the 2–superedges as collections of these stage groups,

$$E = \big\{\{A_1, A_2\},\ \{A_2, A_3\}\big\} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

Then $\text{SHG}^{(2)} = (V, E)$ is a 2–SuperHyperGraph (Definition 1.7). Intuitively, A_1 captures the "diagnostic/antibiotic initiation" stage, A_2 the "hemodynamic support" stage, and A_3 an alternative consolidation stage; the superedges encode permissible stage transitions in a care pathway.

1.2| Medical Knowledge Graph

A *Knowledge Graph* is a graph-based representation that encodes facts as entities and their relations, supporting reasoning, semantic search, and knowledge-driven applications [20, 21, 22, 23, 24]. Extended concepts such as the *Knowledge HyperGraph* [25, 26, 27, 28] and the *Knowledge SuperHyperGraph* [29] have also been proposed in the literature. A *Medical Knowledge Graph* adapts this paradigm to the medical domain, modeling entities such as diseases, symptoms, drugs, and procedures, and linking them through clinically meaningful relations to support clinical reasoning and decision making [30, 31, 32, 33, 34, 35].

Definition 1.9 (Medical Knowledge Graph)**.** (cf.[30, 31, 32]) A *Medical Knowledge Graph* (MKG) is a typed, directed graph that represents medical knowledge as *triples* of the form

$$\langle h, r, t\rangle \in \mathcal{E} \times \mathcal{R} \times \mathcal{E},$$

where $h, t \in \mathcal{E}$ are *entities* (e.g., Disease, Symptom, Drug/MedicinalProduct, Procedure, LaboratoryExam, Gene, MedicalDepartment) and $r \in \mathcal{R}$ is a *typed relation* (e.g., has_symptom, treats, contraindicated_with, diagnosed_by, associated_gene). An MKG thus encodes domain facts as a set $\mathcal{T} \subseteq \mathcal{E} \times \mathcal{R} \times \mathcal{E}$ and may retain provenance (data source) for downstream clinical or research use.

Example 1.10 (Disease–Symptom–Therapy Subgraph)**.** Let entities

$$\mathsf{Gallstone} \in \mathsf{Disease}, \quad \mathsf{Abdominal\ Pain} \in \mathsf{Symptom},$$

$$\mathsf{Ultrasound} \in \mathsf{Radiographic\ Exam}, \quad \mathsf{Ursodeoxycholic\ Acid} \in \mathsf{MedicinalProduct}.$$

A small MKG fragment can be recorded as the triple set

$$\big\{\langle \mathsf{Gallstone}, \mathsf{has\_symptom}, \mathsf{Abdominal\ Pain}\rangle,$$
$$\langle \mathsf{Gallstone}, \mathsf{diagnosed\_by}, \mathsf{Ultrasound}\rangle,$$
$$\langle \mathsf{Ursodeoxycholic\ Acid}, \mathsf{treats}, \mathsf{Gallstone}\rangle\big\}.$$

Such fragments support tasks like clinical decision support or knowledge-enhanced NER/RE.

Example 1.11 (Drug–Interaction and Lab Marker)**.** Let

$$\mathsf{Warfarin}, \mathsf{Aspirin} \in \mathsf{MedicinalProduct}, \quad \mathsf{INR} \in \mathsf{LaboratoryExam}.$$

An MKG can encode pharmacology knowledge as

$$\big\{\langle \mathsf{Warfarin}, \mathsf{interacts\_with}, \mathsf{Aspirin}\rangle,\ \langle \mathsf{Warfarin}, \mathsf{monitored\_by}, \mathsf{INR}\rangle\big\},$$

which is useful for medication safety alerts and therapy monitoring.

2| Main Results

In this section, we present the main contributions of the paper by introducing the definitions and properties of Medical Knowledge HyperGraphs and Medical Knowledge SuperHyperGraphs, and by examining their structural characteristics.

2.1| Medical Knowledge HyperGraph

A Medical Knowledge HyperGraph further generalizes the framework by allowing hyperedges that connect multiple medical entities at once, thereby capturing complex clinical relationships that cannot be expressed through simple triples.

Definition 2.1 (Medical Knowledge HyperGraph (MKHG))**.** Let $\mathcal{E}$ be a finite set of *medical entities* (e.g., diseases, symptoms, drugs, procedures, laboratory exams) and let $\mathcal{C}$ be a set of *entity categories*. A typing map $\mathrm{typ} : \mathcal{E} \to \mathcal{C}$ assigns a category to each entity.

Let $\mathcal{R}$ be a finite set of *relation types* equipped with

- an *arity* function $\mathrm{ar} : \mathcal{R} \to \mathbb{N}_{\geq 1}$, and
- a *signature* function $\mathrm{sig} : \mathcal{R} \to \bigcup_{k\geq 1} \mathcal{C}^k$,

such that if $\mathrm{ar}(r) = k$ then $\mathrm{sig}(r) = (c_1, \ldots, c_k) \in \mathcal{C}^k$ specifies the required category for each argument position.

A *Medical Knowledge HyperGraph (MKHG)* is a tuple

$$\mathcal{K} = (\mathcal{E}, \mathcal{R}, \mathcal{H}, \mathrm{typ}, \mathrm{ar}, \mathrm{sig})$$

where $\mathcal{H}$ is a finite set of *directed, typed hyperedges* of the form

$$h = (r, (e_1, \ldots, e_k)) \quad \text{with} \quad r \in \mathcal{R},\ k = \mathrm{ar}(r),\ e_i \in \mathcal{E},$$

subject to the typing constraint $\mathrm{typ}(e_i) = \mathrm{sig}(r)_i$ for each $i = 1, \ldots, k$. The tuple order encodes direction/argument roles; when $k = 2$ this reduces to a directed, typed edge.

Example 2.2 (A multi-entity clinical rule as a hyperedge)**.** Let $\mathcal{E}$ contain the entities

$$\mathsf{Sepsis} \in \mathsf{Disease}, \quad \mathsf{Fever}, \mathsf{Hypotension} \in \mathsf{Symptom}, \quad \mathsf{SerumLactate} \in \mathsf{LaboratoryExam}.$$

Define a relation type $r = \mathsf{diagnostic\_criterion}$ with

$$\mathrm{ar}(r) = 4, \qquad \mathrm{sig}(r) = (\mathsf{Disease}, \mathsf{Symptom}, \mathsf{Symptom}, \mathsf{LaboratoryExam}).$$

Then the hyperedge

$$h = \big(\mathsf{diagnostic\_criterion}, (\mathsf{Sepsis}, \mathsf{Fever}, \mathsf{Hypotension}, \mathsf{SerumLactate})\big) \in \mathcal{H}$$

encodes, in a single typed object, a rule template that simultaneously relates a disease to multiple symptoms and a lab marker. Such $k > 2$ relations cannot be represented by a single binary triple without factorization.

Example 2.3 (Pharmacogenomic Dosing Rule (Drug–Gene–Disease, k=3))**.** Let the entity set and categories be

$$\mathcal{E} = \{\mathsf{Clopidogrel}, \mathsf{Warfarin}, \mathsf{ACS}, \mathsf{AF}, \mathsf{CYP2C19\_LOF}, \mathsf{VKORC1\_A}\}, \qquad \mathcal{C} = \{\mathsf{Drug}, \mathsf{Disease}, \mathsf{GeneVariant}\}.$$

Define the typing map

$$\mathsf{Clopidogrel}, \mathsf{Warfarin} \in \mathsf{Drug}, \quad \mathsf{ACS}\ (\text{acute coronary syndrome}),\ \mathsf{AF}\ (\text{atrial fibrillation}) \in \mathsf{Disease},$$
$$\mathsf{CYP2C19\_LOF},\ \mathsf{VKORC1\_A} \in \mathsf{GeneVariant}.$$

Introduce a relation type $r = \mathsf{requires\_genotype\_guided\_dosing}$ with

$$\mathrm{ar}(r) = 3, \qquad \mathrm{sig}(r) = (\mathsf{Drug}, \mathsf{GeneVariant}, \mathsf{Disease}).$$

Then the hyperedge set

$$\mathcal{H} = \Big\{ \big(r, (\mathsf{Clopidogrel}, \mathsf{CYP2C19\_LOF}, \mathsf{ACS})\big),\ \big(r, (\mathsf{Warfarin}, \mathsf{VKORC1\_A}, \mathsf{AF})\big) \Big\}$$

is admissible because, for each argument position $i = 1, 2, 3$, the type $\mathrm{typ}(e_i)$ matches the required category $\mathrm{sig}(r)_i$. The tuple order encodes direction/roles: *(drug, genotype, indication)*. This k>2 structure captures pharmacogenomic dosing rules that cannot be represented by a single binary triple without factorization.

Example 2.4 (Acute Stroke Therapy Eligibility (Disease–Imaging–TimeWindow–Procedure, k=4))**.** Let

$$\mathcal{E} = \{\mathsf{IschemicStroke},\ \mathsf{NoncontrastCT},\ \mathsf{CTA},\ \mathsf{Within4\_5h},\ \mathsf{Within24h},\ \mathsf{IVT\_Alteplase},\ \mathsf{EVT\_Thrombectomy}\}.$$

Use categories

$$\mathcal{C} = \{\mathsf{Disease},\ \mathsf{Imaging},\ \mathsf{TimeWindow},\ \mathsf{Procedure}\},$$

and typing

$$\mathsf{IschemicStroke} \in \mathsf{Disease}, \quad \mathsf{NoncontrastCT}, \mathsf{CTA} \in \mathsf{Imaging},$$
$$\mathsf{Within4\_5h}, \mathsf{Within24h} \in \mathsf{TimeWindow}, \quad \mathsf{IVT\_Alteplase}, \mathsf{EVT\_Thrombectomy} \in \mathsf{Procedure}.$$

Define the relation $r = \mathsf{therapy\_eligibility}$ by

$$\mathrm{ar}(r) = 4, \qquad \mathrm{sig}(r) = (\mathsf{Disease},\ \mathsf{Imaging},\ \mathsf{TimeWindow},\ \mathsf{Procedure}).$$

Consider the hyperedges

$$h_1 = \big(r, (\mathsf{IschemicStroke}, \mathsf{NoncontrastCT}, \mathsf{Within4\_5h}, \mathsf{IVT\_Alteplase})\big),$$
$$h_2 = \big(r, (\mathsf{IschemicStroke}, \mathsf{CTA}, \mathsf{Within24h}, \mathsf{EVT\_Thrombectomy})\big).$$

Both h_1 and h_2 satisfy the typing constraint $\mathrm{typ}(e_i) = \mathrm{sig}(r)_i$ for all $i = 1, \ldots, 4$. Thus an MKHG

$$\mathcal{K} = (\mathcal{E}, \{r\}, \{h_1, h_2\}, \mathrm{typ}, \mathrm{ar}, \mathrm{sig})$$

encodes two clinically meaningful 4–ary relations linking a disease to an imaging prerequisite, a time window, and a specific reperfusion procedure.

Theorem 2.5 (Expressiveness: MKHG generalizes hypergraphs and medical knowledge graphs)**.**

*(i) (*Hypergraph embedding*) Every finite hypergraph $H = (V, E)$ can be represented by an MKHG $\mathcal{K}_H$ so that H is recovered from $\mathcal{K}_H$ by forgetting types, labels, and argument order.*

*(ii) (*Medical Knowledge Graph embedding*) Every Medical Knowledge Graph $G = (\mathcal{E}, \mathcal{R}, \mathcal{T})$ consisting of typed binary triples $\langle h, r, t\rangle$ is the k=2 fragment of an MKHG $\mathcal{K}_G$ obtained by taking* $\mathrm{ar}(r) = 2$ *and mapping each triple to a typed hyperedge.*

Proof: **(i) Hypergraph embedding.** Let $H = (V, E)$ be a finite hypergraph. Set $\mathcal{E} = V$ and choose a single category $\mathcal{C} = \{\star\}$ with $\mathrm{typ}(v) = \star$ for all $v \in V$. For each hyperedge $S \in E$ with $|S| = k \geq 1$, assign a distinct relation label r_S with

$$\mathrm{ar}(r_S) = k, \qquad \mathrm{sig}(r_S) = (\underbrace{\star, \ldots, \star}_{k \text{ times}}).$$

Fix any total order $\prec$ on V and list the elements of S as $(v_1, \ldots, v_k)$ in increasing $\prec$-order. Define the hyperedge

$$h_S = \big(r_S, (v_1, \ldots, v_k)\big) \in \mathcal{H}.$$

Let $\mathcal{K}_H = (\mathcal{E}, \mathcal{R}, \mathcal{H}, \mathrm{typ}, \mathrm{ar}, \mathrm{sig})$ with $\mathcal{R} = \{r_S \mid S \in E\}$. By construction each h_S satisfies the typing constraint, hence $\mathcal{K}_H$ is an MKHG.

To *recover* H from $\mathcal{K}_H$, apply the forgetful map that (a) discards labels and types, (b) replaces each ordered tuple by its underlying set. This yields the edge set $\{\, \{v_1, \ldots, v_k\} \mid (r_S, (v_1, \ldots, v_k)) \in \mathcal{H} \,\} = E$ on vertex set V, i.e., the original hypergraph (up to the harmless choice of $\prec$).

(ii) Medical Knowledge Graph embedding. Let $G = (\mathcal{E}, \mathcal{R}, \mathcal{T})$ be an MKG of typed binary triples. Retain the same entity set $\mathcal{E}$ and their categories $\mathcal{C}$ with the typing map typ. For each $r \in \mathcal{R}$ set

$$\mathrm{ar}(r) = 2, \qquad \mathrm{sig}(r) = (c_{\mathrm{head}}, c_{\mathrm{tail}}),$$

matching the declared domain/codomain categories of r in G. Define the hyperedge set

$$\mathcal{H} = \big\{\, \big(r, (h, t)\big) \;\big|\; \langle h, r, t\rangle \in \mathcal{T} \,\big\}.$$

Then $\mathcal{K}_G = (\mathcal{E}, \mathcal{R}, \mathcal{H}, \mathrm{typ}, \mathrm{ar}, \mathrm{sig})$ is an MKHG in which every triple is a length-2 hyperedge obeying the signature constraint. Conversely, the *binary slice* of any MKHG (all hyperedges with $k = 2$) reproduces a medical knowledge graph by reading $(r, (h, t))$ as the triple $\langle h, r, t\rangle$. □

2.2| Medical Knowledge SuperHyperGraph

A Medical Knowledge SuperHyperGraph introduces hierarchical layers via iterated powersets, enabling the representation of multi-level medical relations and generalizing graphs, hypergraphs, and typed medical knowledge models.

Definition 2.6 (Flattening of supervertices)**.** Let E_0 be a finite base set (of *atoms*). For $n \in \mathbb{N}_0$ and $A \in \mathcal{P}^n(E_0)$, define the *flattening* map

$$\mathrm{Flat}_0(e) \coloneqq \{e\} \quad (e \in E_0), \qquad \mathrm{Flat}_{n+1}(A) \coloneqq \bigcup_{B \in A} \mathrm{Flat}_n(B).$$

Thus $\mathrm{Flat}_n(A) \subseteq E_0$ is the set of all atoms occurring inside A at level n.

Definition 2.7 (Medical Knowledge SuperHyperGraph (MKSHG))**.** Let

- E_0 be a finite set of *medical atoms* (e.g., diseases, symptoms, drugs, procedures, laboratory exams);
- $\mathcal{C}$ be a finite set of *categories* and $\mathrm{typ} : E_0 \to \mathcal{C}$ a typing map;
- $\mathcal{R}$ a finite set of *relation types* equipped with an arity map $\mathrm{ar} : \mathcal{R} \to \mathbb{N}_{\geq 1}$ and a signature map $\mathrm{sig} : \mathcal{R} \to \bigcup_{k \geq 1} \mathcal{C}^k$ so that, for $r \in \mathcal{R}$ with $k = \mathrm{ar}(r)$, we have $\mathrm{sig}(r) = (c_1, \ldots, c_k)$.

Fix $n \in \mathbb{N}_0$. A *Medical Knowledge SuperHyperGraph of level n* is a tuple

$$\mathcal{K}^{(n)} = (E_0, \mathcal{C}, \mathrm{typ}, \mathcal{R}, \mathrm{ar}, \mathrm{sig}, V, \mathcal{H})$$

where $V \subseteq \mathcal{P}^n(E_0)$ is a finite set of *n–supervertices*, and $\mathcal{H}$ is a finite set of *directed, typed superhyperedges* of the form

$$h = \big(r, (A_1, \ldots, A_k)\big) \quad \text{with} \quad r \in \mathcal{R},\ k = \mathrm{ar}(r),\ A_i \in V,$$

subject to the *admissibility constraint*

$$\forall i \in \{1, \ldots, k\}\ \ \forall e \in \mathrm{Flat}_n(A_i)\ :\ \mathrm{typ}(e) = \mathrm{sig}(r)_i.$$

When $k = 2$, a superhyperedge is a typed directed edge $(r, (A_1, A_2))$; when $k > 2$, it encodes a k–ary clinical relation among supervertices.

Example 2.8 (A level–2 clinical pathway constraint)**.** Let E_0 contain

$$\mathsf{Sepsis} \in \mathsf{Disease}, \quad \mathsf{Fever}, \mathsf{Hypotension} \in \mathsf{Symptom},$$

$$\mathsf{SerumLactate} \in \mathsf{LaboratoryExam}, \quad \mathsf{IVFluids}, \mathsf{Vasopressor} \in \mathsf{Procedure}.$$

At level $n = 2$ take the supervertices

$$A = \big\{\{\mathsf{Sepsis}\}\big\}, \quad B = \big\{\{\mathsf{Fever}, \mathsf{Hypotension}\}\big\},$$

$$C = \big\{\{\mathsf{SerumLactate}\}\big\}, \quad D = \big\{\{\mathsf{IVFluids}, \mathsf{Vasopressor}\}\big\} \in \mathcal{P}^2(E_0).$$

Define $r = \mathsf{pathway\_step}$ with $\mathrm{ar}(r) = 4$ and

$$\mathrm{sig}(r) = (\mathsf{Disease}, \mathsf{Symptom}, \mathsf{LaboratoryExam}, \mathsf{Procedure}).$$

Then $h = (r, (A, B, C, D))$ is an admissible superhyperedge because $\mathrm{Flat}_2(A) = \{\mathsf{Sepsis}\}$, $\mathrm{Flat}_2(B) = \{\mathsf{Fever}, \mathsf{Hypotension}\}$, $\mathrm{Flat}_2(C) = \{\mathsf{SerumLactate}\}$, $\mathrm{Flat}_2(D) = \{\mathsf{IVFluids}, \mathsf{Vasopressor}\}$, and the types match the signature positions.

Example 2.9 (Level-1 MKSHG: Multimodal COVID-19 Triage Rule)**.** Let the finite set of medical atoms be

$$E_0 = \{\mathsf{COVID19}, \mathsf{Influenza}, \mathsf{Fever}, \mathsf{Cough}, \mathsf{Dyspnea}, \mathsf{PCR}, \mathsf{Antigen}, \mathsf{ChestXray}, \mathsf{ChestCT}\}.$$

Assume the typing map $\mathrm{typ} : E_0 \to \mathcal{C}$ with categories

$$\mathcal{C} = \{\mathsf{Disease}, \mathsf{Symptom}, \mathsf{LaboratoryExam}, \mathsf{Imaging}\}$$

given by

$$\mathsf{COVID19}, \mathsf{Influenza} \in \mathsf{Disease}, \quad \mathsf{Fever}, \mathsf{Cough}, \mathsf{Dyspnea} \in \mathsf{Symptom},$$

$$\mathsf{PCR}, \mathsf{Antigen} \in \mathsf{LaboratoryExam}, \quad \mathsf{ChestXray}, \mathsf{ChestCT} \in \mathsf{Imaging}.$$

Fix $n = 1$ and define the set of supervertices $V \subseteq \mathcal{P}^1(E_0) = \mathcal{P}(E_0)$ by

$$A = \{\mathsf{COVID19}\}, \quad B = \{\mathsf{Fever}, \mathsf{Cough}, \mathsf{Dyspnea}\},$$

$$C = \{\mathsf{PCR}\}, \quad D = \{\mathsf{ChestXray}, \mathsf{ChestCT}\}.$$

Let the relation type $r = \mathsf{triage\_criterion}$ have arity and signature

$$\mathrm{ar}(r) = 4, \qquad \mathrm{sig}(r) = (\mathsf{Disease}, \mathsf{Symptom}, \mathsf{LaboratoryExam}, \mathsf{Imaging}).$$

The superhyperedge

$$h = \big(r, (A, B, C, D)\big)$$

is *admissible* since

$$\mathrm{Flat}_1(A) = \{\mathsf{COVID19}\} \subseteq \mathsf{Disease},$$

$$\mathrm{Flat}_1(B) = \{\mathsf{Fever}, \mathsf{Cough}, \mathsf{Dyspnea}\} \subseteq \mathsf{Symptom},$$

$$\mathrm{Flat}_1(C) = \{\mathsf{PCR}\} \subseteq \mathsf{LaboratoryExam},$$

$$\mathrm{Flat}_1(D) = \{\mathsf{ChestXray}, \mathsf{ChestCT}\} \subseteq \mathsf{Imaging}.$$

Interpretation: a single typed rule links a target disease to a *set* of symptoms, a lab test choice, and an imaging panel. This k>2 relation cannot be captured by one binary triple without factorization.

Example 2.10 (Level-3 MKSHG: Precision Oncology Regimen Selection)**.** Let the atom set include

$$E_0 = \{\mathsf{NSCLC},\ \mathsf{EGFR\_Ex19del},\ \mathsf{ALK\_fusion},\ \mathsf{Osimertinib},\ \mathsf{Alectinib},\ \mathsf{Carboplatin},\ \mathsf{Pemetrexed}\}.$$

Use categories

$$\mathcal{C} = \{\mathsf{Disease},\ \mathsf{GeneVariant},\ \mathsf{Drug}\},$$

and typing

$$\mathsf{NSCLC} \in \mathsf{Disease}, \quad \mathsf{EGFR\_Ex19del}, \mathsf{ALK\_fusion} \in \mathsf{GeneVariant},$$
$$\mathsf{Osimertinib}, \mathsf{Alectinib}, \mathsf{Carboplatin}, \mathsf{Pemetrexed} \in \mathsf{Drug}.$$

Fix $n = 3$. Recall that $\mathcal{P}^3(E_0) = \mathcal{P}\big(\mathcal{P}^2(E_0)\big)$; its elements are collections of level-2 supervertices. Define three level-3 supervertices $V \subseteq \mathcal{P}^3(E_0)$:

$$A = \big\{\{\{\mathsf{NSCLC}\}\}\big\}, \quad B = \big\{\{\{\mathsf{EGFR\_Ex19del}\}\},\ \{\{\mathsf{ALK\_fusion}\}\}\big\},$$
$$C = \big\{\{\{\mathsf{Osimertinib}\}\},\ \{\{\mathsf{Alectinib}\}\},\ \{\{\mathsf{Carboplatin}, \mathsf{Pemetrexed}\}\}\big\}.$$

Let $r = \mathsf{personalized\_regimen}$ with

$$\mathrm{ar}(r) = 3, \qquad \mathrm{sig}(r) = (\mathsf{Disease},\ \mathsf{GeneVariant},\ \mathsf{Drug}).$$

Then the superhyperedge

$$h = \big(r, (A, B, C)\big) \in \mathcal{H}$$

satisfies the admissibility constraint because

$$\mathrm{Flat}_3(A) = \{\mathsf{NSCLC}\} \subseteq \mathsf{Disease}, \quad \mathrm{Flat}_3(B) = \{\mathsf{EGFR\_Ex19del}, \mathsf{ALK\_fusion}\} \subseteq \mathsf{GeneVariant},$$
$$\mathrm{Flat}_3(C) = \{\mathsf{Osimertinib}, \mathsf{Alectinib}, \mathsf{Carboplatin}, \mathsf{Pemetrexed}\} \subseteq \mathsf{Drug}.$$

Interpretation: a hierarchical object C captures alternative *regimen sets* (single agents or combinations), linked to a tumor class A and biomarker group B. Using $n = 3$ permits packaging alternative, stageable regimens while preserving clean typing at the atom level.

Theorem 2.11 (Expressiveness)**.** *For any fixed $n \in \mathbb{N}_0$, the class of level–n Medical Knowledge SuperHyperGraphs (MKSHGs) strictly generalizes:*

(i) finite n–SuperHyperGraphs;

(ii) Medical Knowledge HyperGraphs (MKHGs);

(iii) Medical Knowledge Graphs (MKGs) of typed binary triples.

Proof: **(i) SuperHyperGraph ⇒ MKSHG.** Let $\mathrm{SHG}^{(n)} = (V, E)$ be an n–SuperHyperGraph on the finite base V_0, so $V \subseteq \mathcal{P}^n(V_0)$ and $E \subseteq \mathcal{P}(V) \setminus \{\emptyset\}$. Construct an MKSHG $\Phi(\mathrm{SHG}^{(n)})$ as follows:

$$E_0 \coloneqq V_0, \quad \mathcal{C} \coloneqq \{\star\}, \quad \mathrm{typ}(e) \equiv \star, \quad V \text{ as given.}$$

For each $S = \{A_1, \ldots, A_k\} \in E$ (choose any fixed ordering of its elements), introduce a fresh relation r_S with

$$\mathrm{ar}(r_S) = k, \qquad \mathrm{sig}(r_S) = (\underbrace{\star, \ldots, \star}_{k}).$$

Add the superhyperedge $h_S = (r_S, (A_1, \ldots, A_k))$ to $\mathcal{H}$. By construction, the admissibility constraint holds trivially because $\mathrm{typ} \equiv \star$. Define

$$\Phi(\mathrm{SHG}^{(n)}) = (E_0, \mathcal{C}, \mathrm{typ}, \{r_S\}_{S \in E}, \mathrm{ar}, \mathrm{sig}, V, \mathcal{H}).$$

The *forgetful projection* U that erases relation labels and tuple order,

$$U: \ (r, (A_1, \ldots, A_k)) \longmapsto \{A_1, \ldots, A_k\},$$

sends $\Phi(\mathrm{SHG}^{(n)})$ back to (V, E) because each r_S is unique to S. Hence every n–SuperHyperGraph embeds in a level–n MKSHG.

(ii) MKHG ⇒ MKSHG. Let $\mathcal{K} = (\mathcal{E}, \mathcal{R}, \mathcal{H}, \mathrm{typ}, \mathrm{ar}, \mathrm{sig})$ be a Medical Knowledge HyperGraph (typed directed hypergraph on atoms). Take $n = 0$, $E_0 \coloneqq \mathcal{E}$, $V \coloneqq \mathcal{E} \subseteq \mathcal{P}^0(E_0)$, and keep $(\mathcal{R}, \mathrm{ar}, \mathrm{sig})$ unchanged. Each hyperedge $(r, (e_1, \ldots, e_k)) \in \mathcal{H}$ is already an admissible superhyperedge because $\mathrm{Flat}_0(e_i) = \{e_i\}$ and $\mathrm{typ}(e_i) = \mathrm{sig}(r)_i$. Thus MKHGs are precisely the level–0 fragment of MKSHGs.

(iii) MKG $\Rightarrow$ MKSHG. Let $G = (\mathcal{E}, \mathcal{R}, \mathcal{T})$ be a Medical Knowledge Graph of typed binary triples $\langle h, r, t\rangle$ with declared domain/codomain categories (c_1, c_2) for each $r \in \mathcal{R}$. Again set $n = 0$, $E_0 \coloneqq \mathcal{E}$, $V \coloneqq \mathcal{E}$, and keep the same typing map. Define $\mathrm{ar}(r) \coloneqq 2$ and $\mathrm{sig}(r) \coloneqq (c_1, c_2)$. Map each triple to the superhyperedge $(r, (h, t))$. Admissibility is exactly the MKG type constraint. Hence every MKG embeds as the binary slice of a level–0 MKSHG.

Since the embeddings in (i)–(iii) are injective on objects (up to the harmless choice of an ordering in (i)), MKSHGs strictly generalize the three structures. □

3| Conclusion

In this paper, we extended the Medical Knowledge Graph framework using HyperGraphs and SuperHyperGraphs, and investigated its properties. We expect that future research will further explore extensions based on Fuzzy Graphs[36, 37, 38], Intuitionistic Fuzzy Graphs[39, 40], Neutrosophic Graphs[41, 42], and Plithogenic Graphs[43, 44].

Acknowledgments

The authors wish to thank all colleagues and mentors whose feedback and encouragement enriched this work. We are grateful to the community of researchers whose foundational contributions informed our developments. Special appreciation goes to the institutions that offered resources and technical infrastructure throughout this project.

Funding

No external funding or financial support was provided for this study.

Data Availability

This manuscript presents purely conceptual work without empirical data. Scholars interested in these ideas are invited to undertake experimental or case-study research to substantiate and extend the proposed frameworks.

Conflicts of Interest

The authors declare that there are no competing interests concerning the content or publication of this article.

Ethical Approval

This paper involves no human or animal subjects and thus did not require ethics committee review or approval.

Disclaimer

The theoretical models and propositions herein have not yet been subjected to practical validation. Readers should independently verify all citations and be aware that inadvertent inaccuracies may remain. The opinions expressed are those of the authors and do not necessarily represent the views of affiliated organizations.

Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

References

[1] Gao, Y., Zhang, Z., Lin, H., Zhao, X., Du, S., & Zou, C. (2020). Hypergraph learning: Methods and practices. *IEEE transactions on pattern analysis and machine intelligence*, *44*(5), 2548-2566.

[2] Feng, Y., You, H., Zhang, Z., Ji, R., & Gao, Y. (2019). Hypergraph neural networks. *Proceedings of the AAAI conference on artificial intelligence* (pp. 3558-3565). AAAI. https://doi.org/10.1609/aaai.v33i01.33013558

[3] Jose, B. K., & Tuza, Z. (2009). Hypergraph domination and strong independence. *Applicable analysis and discrete mathematics*, *3*(2), 347-358. https://doi.org/10.2298/AADM0902347J

[4] Campoverde Valencia, E. M., Chuisaca Vásquez, J. P., & Becerra Lois, F. Á. (2025). Multineutrosophic analysis of the relationship between survival and business growth in the manufacturing sector of azuay province, 2020–2023, using plithogenic n-superhypergraphs. *Neutrosophic sets and systems*, *84*(1), 341-355. https://digitalrepository.unm.edu/nss_journal/vol84/iss1/28/

[5] Fujita, T., & Smarandache, T. (2025). *Advancing uncertain combinatorics through graphization, hyperization, and uncertainization: Fuzzy, neutrosophic, soft, rough, and beyond*. Biblio Publishing. https://www.researchgate.net/publication/386082978

[6] Méndez Bravo, J. C., Bolanos Piedrahita, C. J., Méndez Bravo, M. A., & Pilacuan-Bonete, L. M. (2025). Integrating smed and Industry 4.0 to optimize processes with plithogenic n-superhypergraphs. *Neutrosophic sets and systems*, *84*(1), 328-340. https://digitalrepository.unm.edu/nss_journal/vol84/iss1/27/

[7] Fujita, T. (2025). Multi-superhypergraph neural networks: A generalization of multi-hypergraph neural networks. *Neutrosophic computing and machine learning*, *39*(1), 328-347.

[8] Mogro Cepeda, Y. V., Riofrío Guevara, M. A., Jácome Mogro, E. J., & Piovanelli Tizano, R. (2024). Impact of irrigation water technification on seven directories of the san juan-patoa river using plithogenic n-superhypergraphs based on environmental indicators in the Canton of Pujilí, 2021. *Neutrosophic sets and systems*, *74*(1), 47-56. https://digitalrepository.unm.edu/nss_journal/vol74/iss1/6/

[9] Berrocal Villegas, S. M., Montalvo Fritas, W., Berrocal Villegas, C. R., Flores Fuentes Rivera, M. Y., Espejo Rivera, R., Bautista Puma, L. D., & Macazana Fernández, D. M. (2025). Using plithogenic n-superhypergraphs to assess the degree of relationship between information skills and digital competencies. *Neutrosophic sets and systems*, *84*(1), 513-524. https://digitalrepository.unm.edu/nss_journal/vol84/iss1/41/

[10] Zhu, S. (2025). Neutrosophic n-superhypernetwork: A new approach for evaluating short video communication effectiveness in media convergence. *Neutrosophic sets and systems*, *85*(1), 1004-1017. https://digitalrepository.unm.edu/nss_journal/vol85/iss1/58

[11] Nacaroglu, Y., Akgunes, N., Pak, S., & Cangul, I. N. (2021). Some graph parameters of power set graphs. *Advances & applications in discrete mathematics*, *26*(2), 211-219. http://dx.doi.org/10.17654/DM026020211

[12] Hjorth, G. (2005). T. Jech. Set theory. The third millennium edition, revised and expanded. Springer-Verlag, Berlin, 2003, viii + 769 pp. *Bulletin of Symbolic Logic*, *11*(2), 243–245. https://doi.org/10.1017/S1079898600003358

[13] Bretto, A. (2013). *Hypergraph theory. An introduction.* Cham: Springer. https://doi.org/10.1007/978-3-319-00080-0

[14] Berge, C. (1984). *Hypergraphs: Combinatorics of finite sets*. Elsevier.

[15] Smarandache, F. (2024). Foundation of superhyperstructure & neutrosophic superhyperstructure. *Neutrosophic sets and systems*, *63*(2024), 367-381. https://doi.org/10.5281/zenodo.10535374

[16] Khali, H. E., GÜNGÖR, G. D., & Zaina, M. A. N. (2022). Neutrosophic superhyper bi-topological spaces: Original notions and new insights. *Neutrosophic Sets and Systems*, *51*(1), 34-45. https://digitalrepository.unm.edu/cgi/viewcontent.cgi?article=2116&context=nss_journal

[17] Smarandache, F. (2020). Extension of hypergraph to n-superhypergraph and to plithogenic n-superhypergraph, and extension of hyperalgebra to n-ary (classical-/neutro-/anti-) hyperalgebra. *Neutrosophic sets and systems, 33*, 290-296. https://www.researchgate.net/publication/343862930

[18] Ghods, M., Rostami, Z., & Smarandache, F. (2022). Introduction to neutrosophic restricted superhypergraphs and neutrosophic restricted superhypertrees and several of their properties. *Neutrosophic sets and systems*, *50*(1), 480-487. https://digitalrepository.unm.edu/cgi/viewcontent.cgi?article=2101&context=nss_journal

[19] Hamidi, M., & Taghinezhad, M. (2023). *Application of superhypergraphs-based domination number in real world*. *Journal of mahani mathematical research, 13*(1), 211-228. https://doi.org/10.22103/jmmr.2023.21203.1415

[20] Wang, X., He, X., Cao, Y., Liu, M., & Chua, T. S. (2019). Kgat: Knowledge graph attention network for recommendation. *Proceedings of the 25th ACM SIGKDD international conference on knowledge discovery & data mining* (pp. 950-958). ACM. https://doi.org/10.1145/3292500.3330989

[21] Li, J., Shomer, H., Ding, J., Wang, Y., Ma, Y., Shah, N., ... & Yin, D. (2023, July). Are message passing neural networks really helpful for knowledge graph completion? *Proceedings of the 61st annual meeting of the association for computational linguistics (volume 1: long papers)* (pp. 10696-10711). Association for Computational Linguistics. https://doi.org/10.18653/v1/2023.acl-long.597

[22] Nickel, M., Murphy, K., Tresp, V., & Gabrilovich, E. (2015). A review of relational machine learning for knowledge graphs. *Proceedings of the IEEE*, *104*(1), 11-33. https://doi.org/10.1109/JPROC.2015.2483592

[23] Liu, Y., Yang, S., Xu, Y., Miao, C., Wu, M., & Zhang, J. (2021). Contextualized graph attention network for recommendation with item knowledge graph. *IEEE Transactions on knowledge and data engineering*, *35*(1), 181-195. https://doi.org/10.1109/TKDE.2021.3082948

[24] Yuan, H., Fan, B., Yang, C., & Li, X. (2025). Graph neural networks for knowledge graph construction: Research progress, agricultural development potential, and future directions. *Smart agriculture*, *7*(2), 41-56. https://doi.org/10.12133/j.smartag.SA202501007

[25] Zhou, X., Hui, B., Zeira, I., Wu, H., & Tian, L. (2023). Dynamic relation learning for link prediction in knowledge hypergraphs: Zhou et al. *Applied intelligence*, *53*(22), 26580-26591. https://doi.org/10.1007/s10489-023-04710-5

[26] Li, M., Shi, X., Qiao, C., Zhang, T., & Jin, H. (2024). *Hyperbolic hypergraph neural networks for multi-relational knowledge hypergraph representation*. https://doi.org/10.48550/arXiv.2412.12158

[27] Fatemi, B., Taslakian, P., Vazquez, D., & Poole, D. (2019). *Knowledge hypergraphs: Prediction beyond binary relations*. https://doi.org/10.48550/arXiv.1906.00137

[28] Bhuyan, B. P., Singh, T. P., Tomar, R., & Ramdane-Cherif, A. (2024). Nesykhg: Neuro-symbolic knowledge hypergraphs. *Procedia computer science*, *235*, 1278-1288. https://doi.org/10.1016/j.procs.2024.04.121

[29] Fujita, T. (2025). Knowledge superhypergraphs, multimodal superhypergraphs, lattice superhypergraphs, and hyperbolic superhypergraphs: Concepts and applications. *Journal of operational and strategic analytics*, *3*(2), 95-119. https://doi.org/10.56578/josa030203

[30] Li, L., Wang, P., Yan, J., Wang, Y., Li, S., Jiang, J., ... & Liu, Y. (2020). Real-world data medical knowledge graph: construction and applications. *Artificial intelligence in medicine*, *103*, 101817. https://doi.org/10.1016/j.artmed.2020.101817

[31] Gong, F., Wang, M., Wang, H., Wang, S., & Liu, M. (2021). SMR: Medical knowledge graph embedding for safe medicine recommendation. *Big data research*, *23*, 100174. https://doi.org/10.1016/j.bdr.2020.100174

[32] Yang, P., Wang, H., Huang, Y., Yang, S., Zhang, Y., Huang, L., ... & Huang, Y. (2024). LMKG: A large-scale and multi-source medical knowledge graph for intelligent medicine applications. *Knowledge-based systems*, *284*, 111323. https://doi.org/10.1016/j.knosys.2023.111323

[33] Wang, X., Zhang, R., Zhao, B., Yao, Y., Zhao, H., & Zhu, X. (2024). Medical knowledge graph completion via fusion of entity description and type information. *Artificial intelligence in medicine*, *151*, 102848. https://doi.org/10.1016/j.artmed.2024.102848

[34] Lan, Y., He, S., Liu, K., Zeng, X., Liu, S., & Zhao, J. (2021). Path-based knowledge reasoning with textual semantic information for medical knowledge graph completion. *BMC medical informatics and decision making*, *21*(9), 335. https://doi.org/10.1186/s12911-021-01622-7

[35] Cheng, B., Zhang, J., Liu, H., Cai, M., & Wang, Y. (2021). Research on medical knowledge graph for stroke. *Journal of healthcare engineering*, *2021*(1), 5531327. https://doi.org/10.1155/2021/5531327

[36] Rosenfeld, A. (1975). Fuzzy graphs. In *Fuzzy sets and their applications to cognitive and decision processes* (pp. 77-95). Academic press. https://doi.org/10.1016/B978-0-12-775260-0.50008-6

[37] Mordeson, J. N., & Nair, P. S. (2012). *Fuzzy graphs and fuzzy hypergraphs* (Vol. 46). Physica.

[38] Al-Hawary, T. (2011). Complete fuzzy graphs. *International journal of mathematical combinatorics*, *4*, 26-34. https://www.researchgate.net/publication/314375949

[39] Sahoo, S., & Pal, M. (2017). Product of intuitionistic fuzzy graphs and degree. *Journal of intelligent & fuzzy systems*, *32*(1), 1059-1067. https://doi.org/10.3233/JIFS-16348

[40] Akram, M., Davvaz, B., & Feng, F. (2013). Intuitionistic fuzzy soft k-algebras. *Mathematics in computer science*, *7*(3), 353-365. https://doi.org/10.1007/s11786-013-0158-5

[41] Broumi, S., Talea, M., Bakali, A., & Smarandache, F. (2016). Single valued neutrosophic graphs. *Journal of New theory*, (10), 86-101. https://dergipark.org.tr/en/pub/jnt/article/381241

[42] Broumi, S., Talea, M., Bakali, A., Smarandache, F., & Kumar, P. K. (2017). Shortest path problem on single valued neutrosophic graphs. *2017 international symposium on networks, computers and communications (ISNCC)* (pp. 1-6). IEEE. https://doi.org/10.1109/ISNCC.2017.8071993

[43] Sultana, F., Gulistan, M., Ali, M., Yaqoob, N., Khan, M., Rashid, T., & Ahmed, T. (2022). A study of plithogenic graphs: Applications in spreading coronavirus disease (covid-19) globally. *Journal of ambient intelligence and humanized computing*, *14*, 13139–13159. https://doi.org/10.1007/s12652-022-03772-6

[44] Kandasamy, W. V., Ilanthenral, K., & Smarandache, F. (2020). *Plithogenic graphs*. https://www.researchgate.net/publication/340438419